Stability of Nonlinear Control Systems Under Intermittent Information with Applications to Multi-Agent Robotics

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ACROSS Colloquia
26.10.2012.

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Outline

1. Introduction
   - Motivation
   - Intermittent Information
   - Recent Ideas
2. Stability
   - Problem Formulation
   - Approach
   - Numerical Results
3. Optimal Intermittent Fdbk
   - Motivation
   - Approach
   - Numerical Results
4. Summary

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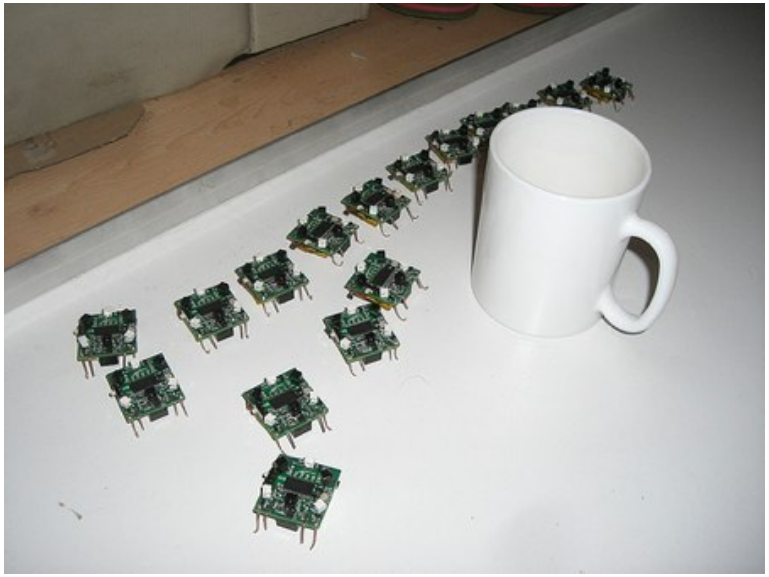
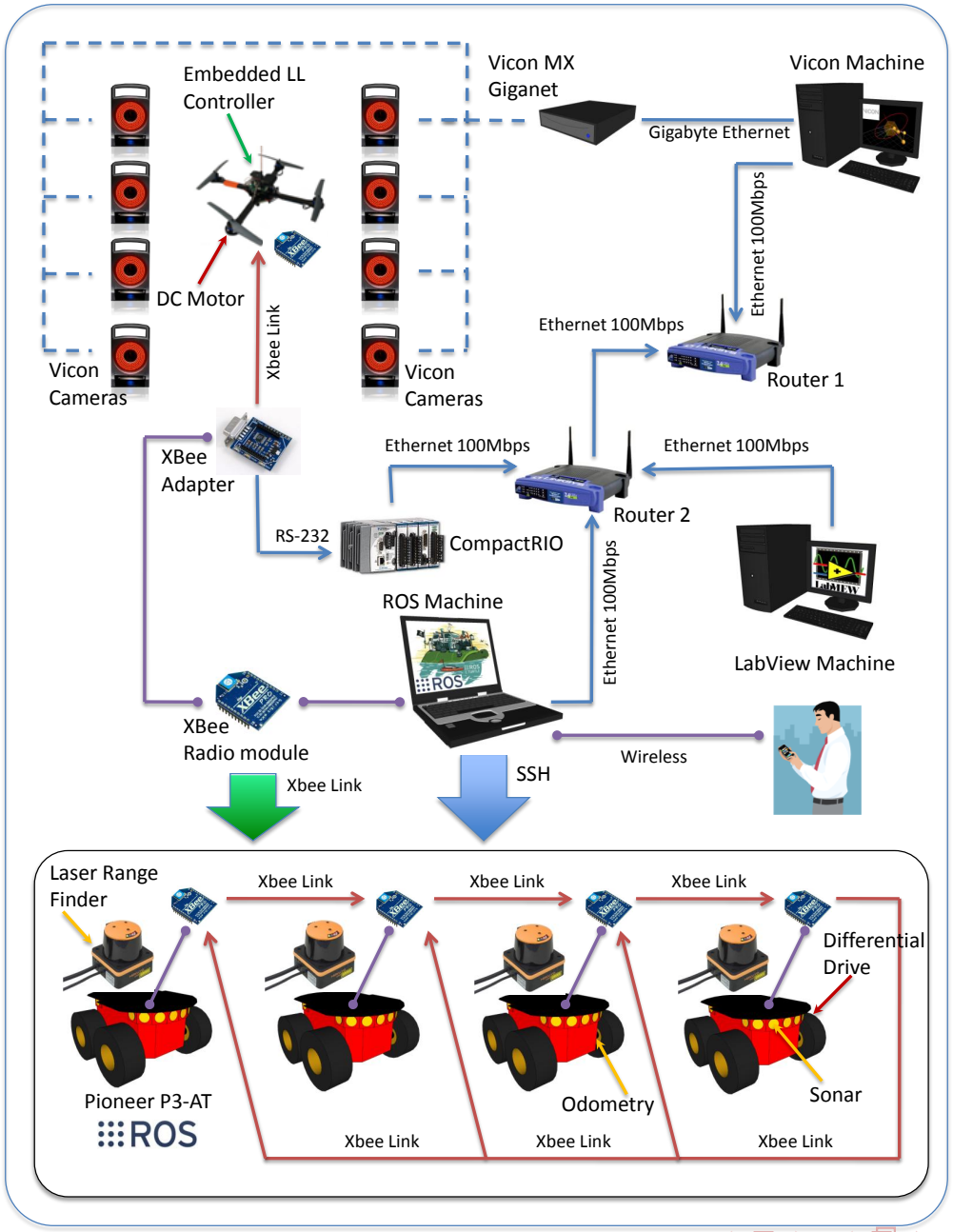
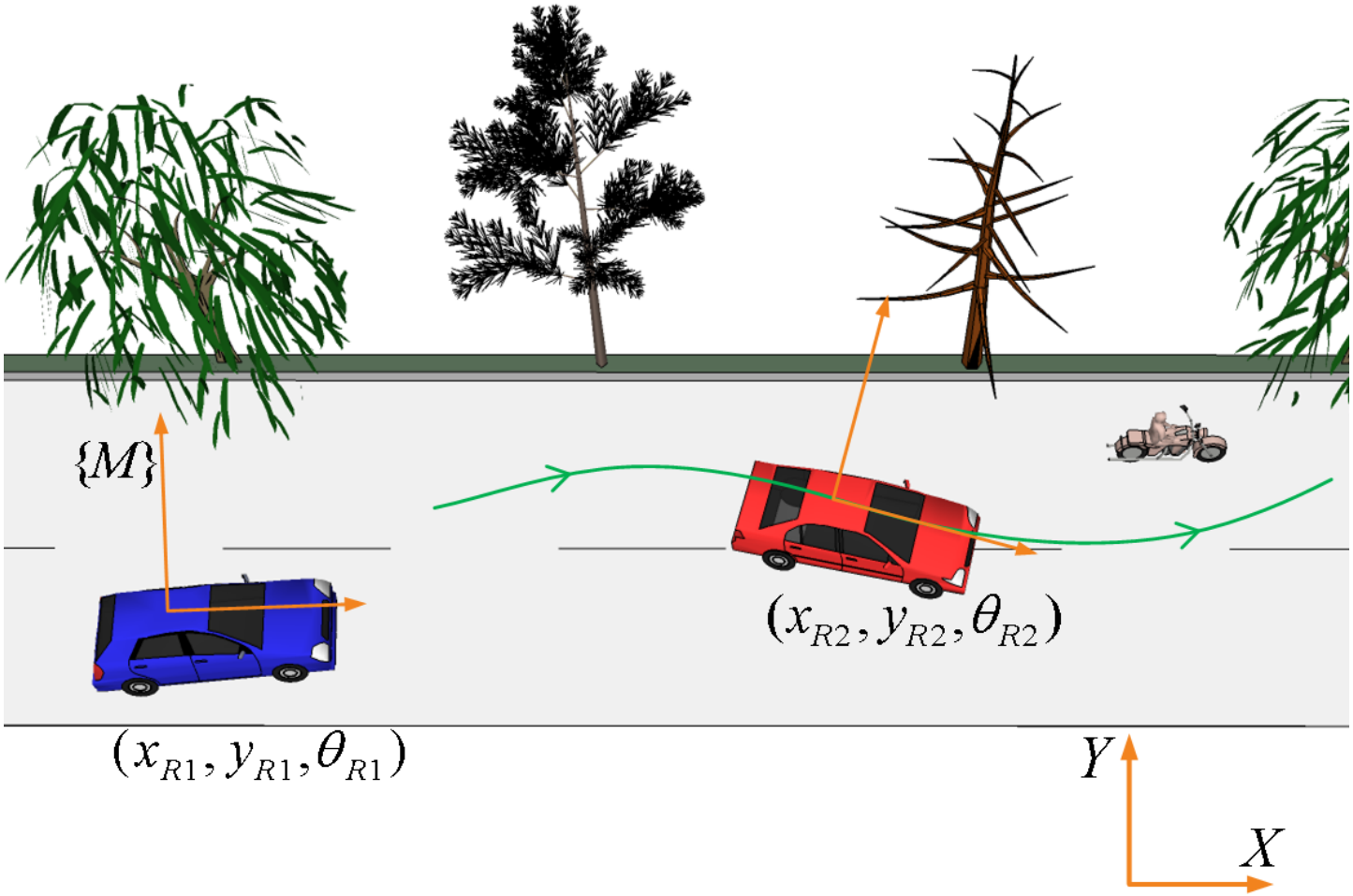
Multi-Agent Systems



Systems of Systems (Heterogeneity)



Illustrative Example



What Do We Mean by Intermittent Information?

- How often should **up-to-date information** about systems be collected and sent to **neighbors**?

Intrinsic Properties

- packet collisions, sampling period, network throughput, scheduling protocols, occlusions of sensors, a limited communication/sensing range, etc.

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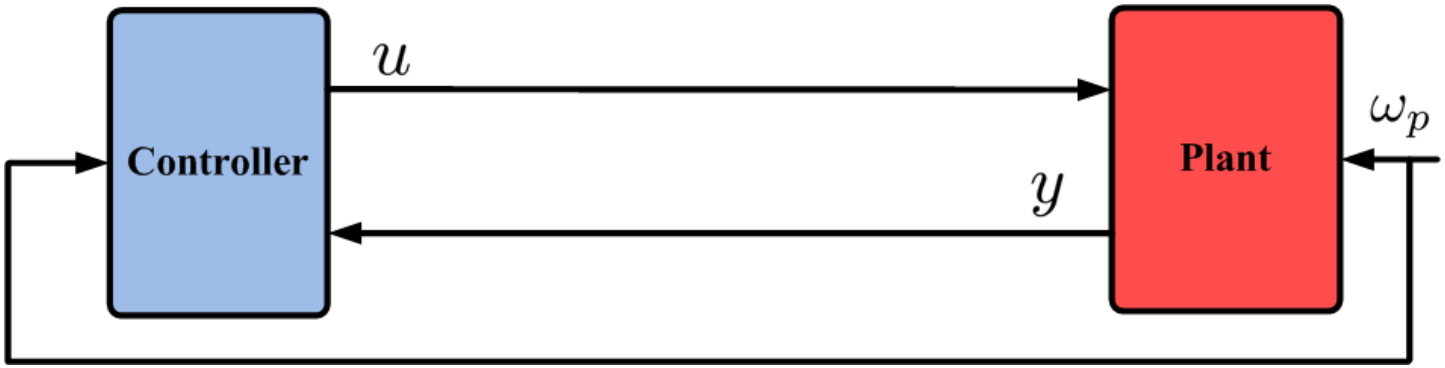
Intermittent Feedback

- a **user-designed** property of a system
- the goal is to **decrease energy consumption** as well as **processing** and **sensing requirements**
- **flexibility** and **resource efficiency**

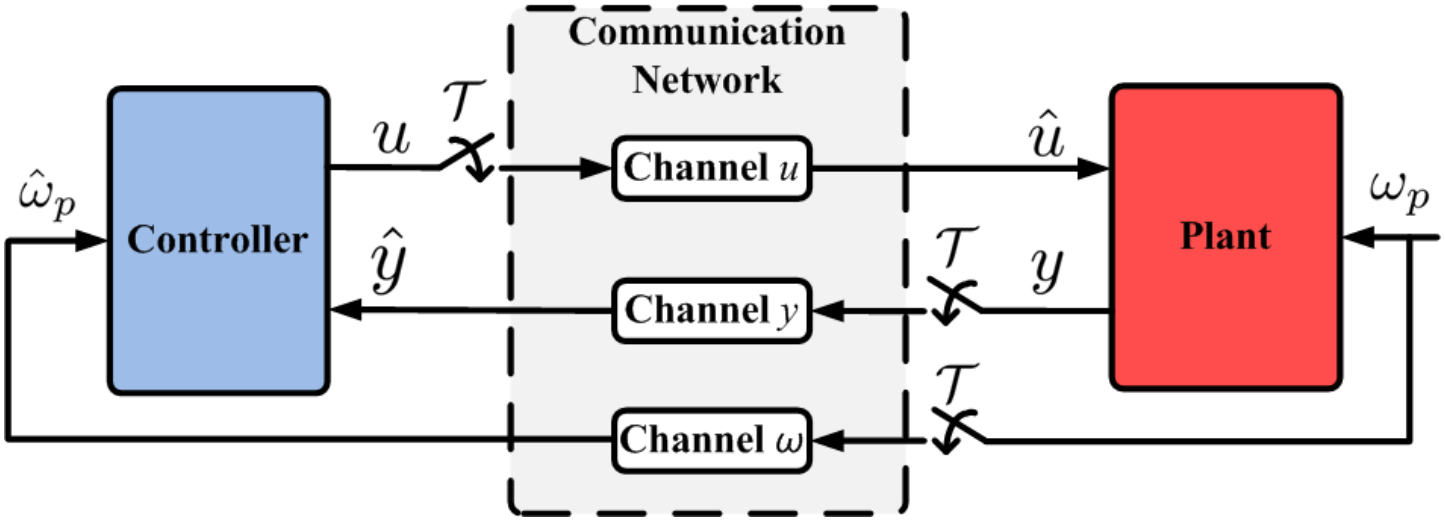
Why Do We Care?

When designing controllers we think of:



Why Do We Care?

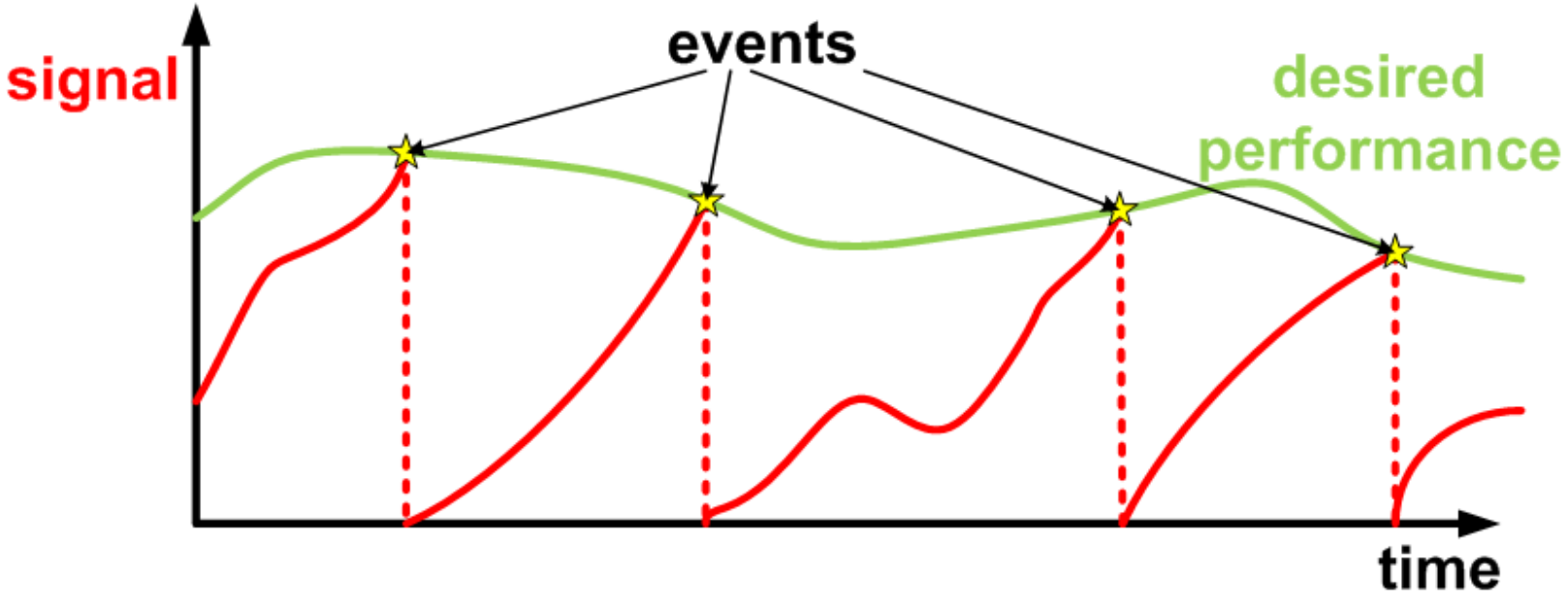
What we really have is:



Event-Triggering vs. Self-Triggering

Event-Triggering

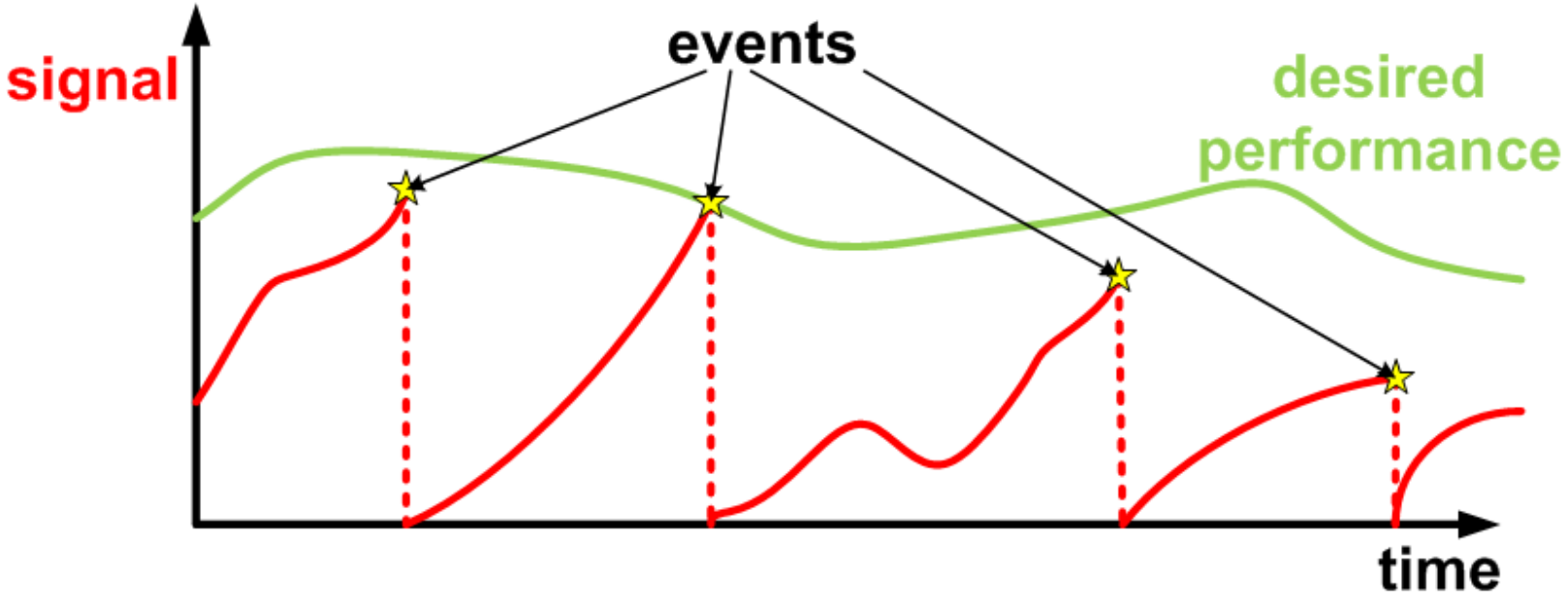
- sampling (i.e., transmission of up-to-date information) is triggered when a **triggering event** occurs



Event-Triggering vs. Self-Triggering

Self-Triggering

- the **current sampling instance** is used to **predict** triggering events and **preclude** undesired performance



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3. Input-to-State Stability (ISS) approaches [Tabuada, 2007], [Anta and Tabuada, 2010], [Lemmon, 2010]; and
4. Other approaches [Estrada and Antsaklis, 2008], [Li et al., 2007], [Tallapragada and Chopra, 2011].

Closed-Loop Systems

- **first** – design a closed-loop system without taking into account communication networks

  – *plant*:

$$\begin{aligned} \dot{x}_p &= f_p(t, x_p, u, \omega_p), \\ y &= g_p(t, x_p), \end{aligned} \tag{1}$$

  – *controller*:

$$\begin{aligned} \dot{x}_c &= f_c(t, x_c, y, \omega_c), \\ u &= g_c(t, x_c) \end{aligned} \tag{2}$$

- **second** – introduce communication networks

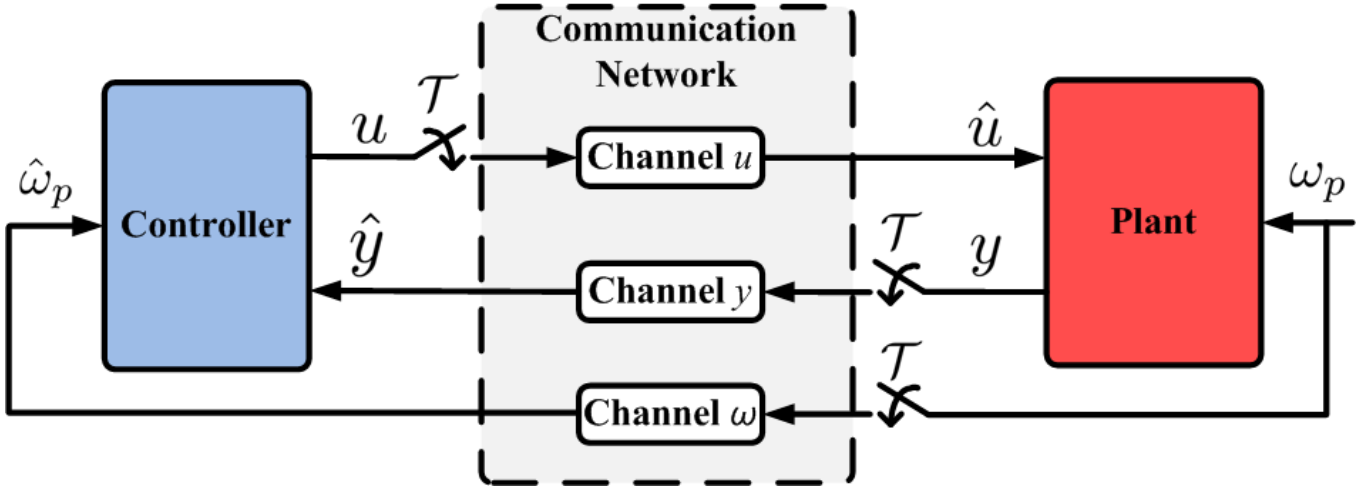
Introducing Communication Networks

- **output error vector**:

$$e := \begin{bmatrix} \hat{y} - y \\ \hat{u} - u \end{bmatrix} =: \begin{bmatrix} e_y \\ e_u \end{bmatrix} \tag{3}$$

- **input error vector**:

$$e_\omega := \hat{\omega}_p - \omega_p \tag{4}$$



Problem Statement

Input-Output Triggering

Based on the value of $\hat{y}$ and $\hat{\omega}_p$ received at t_i, $i \in \mathbb{N}$, find a **time interval** τ_i **until the next transmission** instant of y and ω_p yielding the closed-loop system (1)-(2) **stable in some appropriate sense**.

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1. The design of an **input-output triggered** sampling policy employing the **small-gain theorem**;

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4. The design of a novel method for **computing** $\mathcal{L}_p$-gains over a **finite horizon** by resorting to **convex programming**.

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Our approach does not require construction of storage nor Lyapunov functions.

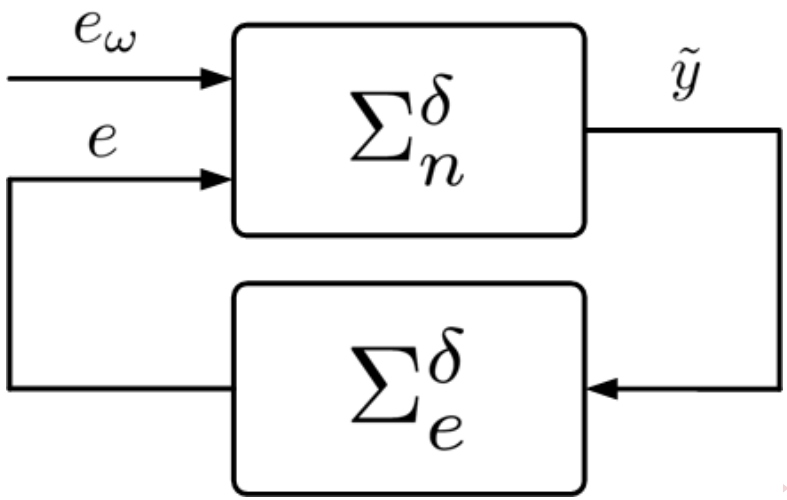
Interconnecting Nominal and Error System I

We write (1)-(2) as the following interconnected hybrid system [Sanfelice, 2007]:

$$\left.\begin{aligned} \dot{x} &= f^{\delta}(t, x, e, e_{\omega}) \\ \dot{e} &= g^{\delta}(t, x, e, e_{\omega}) \end{aligned}\right\} \quad t \in \bigcup_{i \in \mathbb{N}_0} [t_i, t_{i+1}), \tag{5a}$$

$$\left.\begin{aligned} x(t^+) &= x(t) \\ e(t^+) &= h(t, e(t)) \end{aligned}\right\} \quad t \in \mathcal{T}. \tag{5b}$$

where $\delta = \hat{\omega}_p$ and $x = (x_p, x_c)$.



Interconnecting Nominal and Error System II

$$f(t,x,e,\hat{\omega}_p,e_\omega) := \begin{bmatrix} f_p(t,x_p,g_c(t,x_c)+e_u,\hat{\omega}_p-e_\omega) \\ f_c(t,x_c,g_p(t,x_p)+e_y,\hat{\omega}_p) \end{bmatrix},$$

$$h(t_i,e) := \begin{bmatrix} h_y(t_i,e(t_i)) \\ h_u(t_i,e(t_i)) \end{bmatrix},$$

$$g(t,x,e,\hat{\omega}_p,e_\omega) :=$$

$$\begin{bmatrix} \underbrace{\hat{f}_p(t,x_p,x_c,g_p(t,x_p)+e_y,g_c(t,x_c)+e_u,\hat{\omega}_p-e_\omega)}_{\equiv 0 \text{ for zero-order-hold estimation strategy}} - \frac{\partial g_p}{\partial t}(t,x_p) - \frac{\partial g_p}{\partial x_p}(t,x_p) f_p(t,x_p,g_c(t,x_c)+e_u,\hat{\omega}_p-e_\omega) \\ \overbrace{\hat{f}_c(t,x_p,x_c,g_p(t,x_p)+e_y,g_c(t,x_c)+e_u,\hat{\omega}_p)} - \frac{\partial g_c}{\partial t}(t,x_c) - \frac{\partial g_c}{\partial x_c}(t,x_c) f_c(t,x_p,g_p(t,x_p)+e_y,\hat{\omega}_p) \end{bmatrix}$$

$\mathcal{L}_p$-stability of Switched Systems I

Theorem

Consider a hybrid system Σ^δ. Let $K \geq 0$ and $p \in [1, \infty)$. If δ is such that

(i) *There exist constants $\widetilde{K}(\tau_i^\delta)$, $\widetilde{\gamma}(\tau_i^\delta)$ such that*

$$\|y[t_i^\delta, t']\|_p \leq \widetilde{K}(\tau_i^\delta)\|x(t_i^{\delta+})\| + \widetilde{\gamma}(\tau_i^\delta)\|\omega[t_i^\delta, t']\|_p. \tag{6}$$

for all $t' \in [t_i^\delta, t_{i+1}^\delta]$ and all $i \in \mathbb{N}_0$, where $\tau_i^\delta = t_{i+1}^\delta - t_i^\delta$, and such that

$$K_M := \sup_{i \in \mathbb{N}_0} \widetilde{K}(\tau_i^\delta), \tag{7}$$

$$\gamma_M := \sup_{i \in \mathbb{N}_0} \widetilde{\gamma}(\tau_i^\delta), \tag{8}$$

exist, and

$\mathcal{L}_p$-stability of Switched Systems II

Theorem

(ii) *The condition*

$$\sum_{i=1}^{\infty} \|x(t_i^{\delta+})\| \leq K\|x(t_0)\|, \tag{9}$$

holds,

then Σ^δ *is* $\mathcal{L}_p$*-stable from* ω *to* y *with constant* $K_M(K+1)$ *and gain* γ_M *for the given* δ*. For* $p = \infty$*, the same result holds with the constant* $K_M K$ *and gain* γ_M *when (9) is replaced with* $\sup_{i\in\mathbb{N}} \|x(t_i^{\delta+})\| \leq K\|x(t_0)\|$*. In addition, if the state* x *is* $\mathcal{L}_p$ *to* $\mathcal{L}_p$ *detectable, then conditions (i) and (ii) are both sufficient and necessary.*

$\mathcal{L}_p$-stability Over a Finite Horizon for Error System

Theorem

Assume $\delta = \hat{\omega}_p$ is fixed to be constant. Suppose that there exist $A \in \mathcal{A}_{n_e}^+$ such that $\|A\| < \infty$, and a continuous function $\tilde{y} : \mathbb{R} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_\omega} \times \mathbb{R}^{n_\omega} \to \mathbb{R}_+^{n_e}$ such that the output error dynamics in (5a) satisfies

$$\dot{\bar{e}} = \overline{g(t, x, e, \hat{\omega}_p, e_\omega)} \preceq A\bar{e} + \tilde{y}(t, x, \hat{\omega}_p, e_\omega) \tag{10}$$

for all $e \in \mathbb{R}^{n_e}$ and all $(t, x(t), \hat{\omega}_p, e_\omega(t)) \in \mathcal{S}$ provided that $t \in [t_0, t_0 + \tau]$, where $\mathcal{S} \subseteq \mathbb{R} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_\omega} \times \mathbb{R}^{n_\omega}$. Then, the output error system is $\mathcal{L}_p$-stable from $\tilde{y}$ to e over a finite horizon $\tau \geq 0$ for any $p \in [1, \infty]$ with

$$\widetilde{K}_e(\tau) = \left(\frac{\exp(\|A\| p\tau) - 1}{p\|A\|} \right)^{1/p}, \tag{11}$$

$$\widetilde{\gamma}_e(\tau) = \frac{\exp(\|A\|\tau) - 1}{\|A\|}. \tag{12}$$

Input-Output Triggering

- **stabilizing intersampling intervals** are given by

$$\tau_i^* = \frac{1}{\|A_i^\delta\|} \ln\left(\kappa \frac{\|A_i^\delta\|}{\gamma_n^\delta} + 1\right) \Longleftrightarrow \gamma_n^\delta \tilde{\gamma}_e(\tau_i^*) = \kappa, \tag{13}$$

where $\kappa \in (0, 1)$.

Input-Output Triggering

Theorem

For $p \in [1, \infty]$ *assume that*

(i) *There exists* $L \geq 0$ *such that* $\overline{\omega}_p \preceq L\, \mathbf{1}_{n_\omega}$,

(ii) *Theorem 3 holds,*

(iii) Σ_n^δ *is* $\mathcal{L}_p$*-stable from* (e_ω, e) *to* $\tilde{y}$ *for every* $\delta \equiv c$, $c \in \mathcal{P}$, *with gain* γ_n^δ *that has an upper- and lower-bound,*

(iv) *Sampling policy (13) yields* x *that satisfies (9) for given* $\delta : [t_0, \infty) \to \mathcal{P}$, *and*

(v) x *is* $\mathcal{L}_p$ *to* $\mathcal{L}_p$ *detectable from* $(\tilde{y}, e_\omega, e)$.

Then, the closed-loop system (1)-(2) is $\mathcal{L}_p$*-stable from* e_ω *to* (x, e) *for given* δ.

Trajectory Tracking

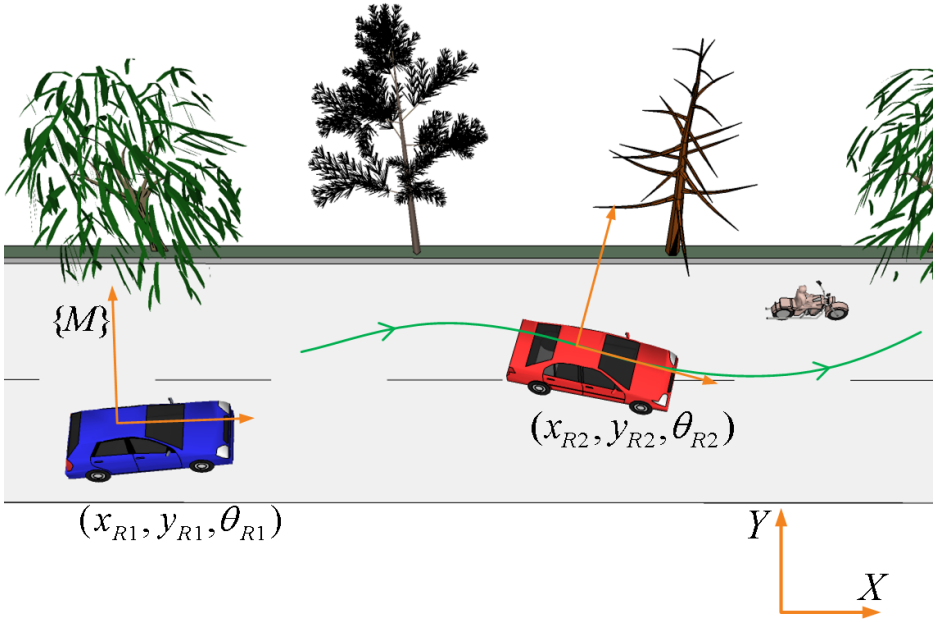
– **tracking error**:

$$\dot{x}_p = \begin{bmatrix} \omega_{R1} x_{p2} - v_{R1} + v_{R2} \cos x_{p3} \\ -\omega_{R1} x_{p1} + v_{R2} \sin x_{p3} \\ \omega_{R2} - \omega_{R1} \end{bmatrix}.$$

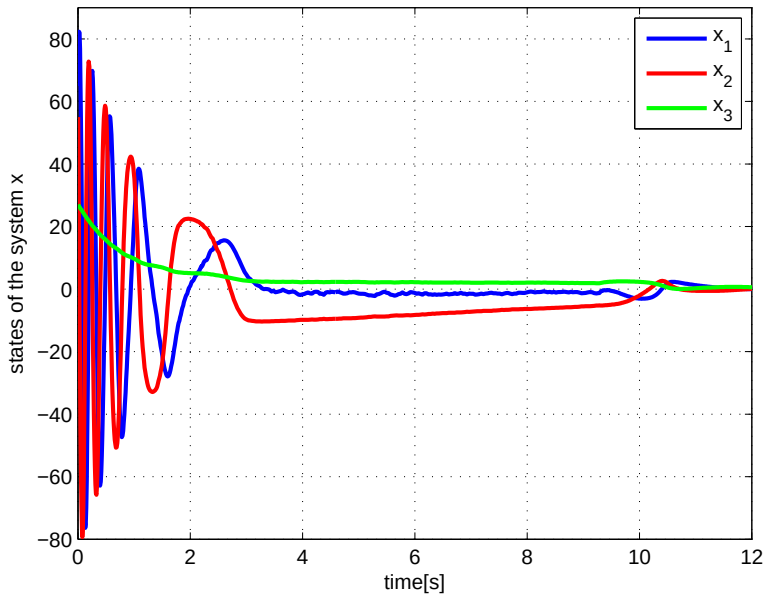
– **controller**:

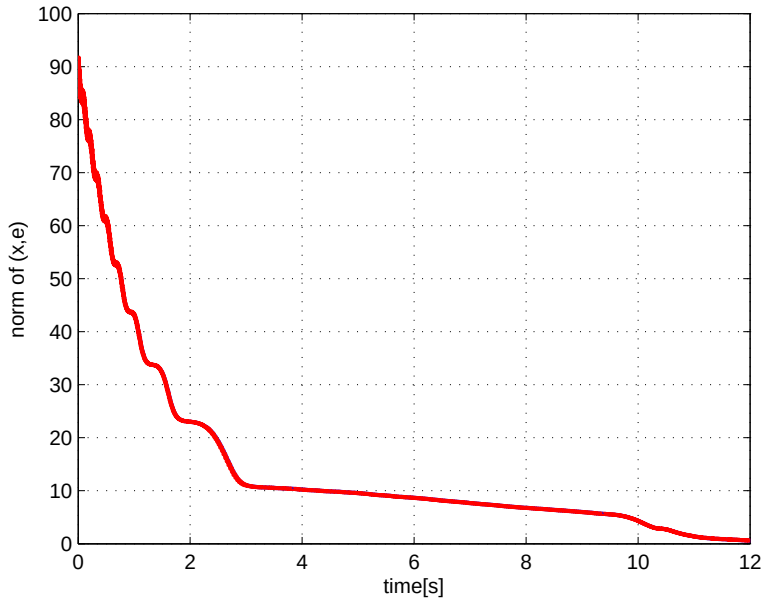
$$v_{R1} = v_{R2} \cos x_{p3} + k_1 x_{p1},$$

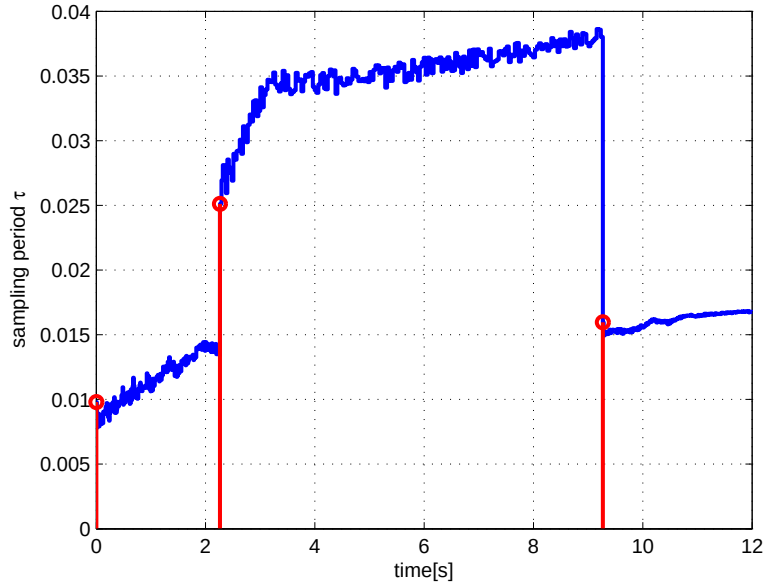
$$\omega_{R1} = \omega_{R2} + k_2 v_{R2} \frac{\sin x_{p3}}{x_{p3}} x_{p2} + k_3 x_{p3}.$$

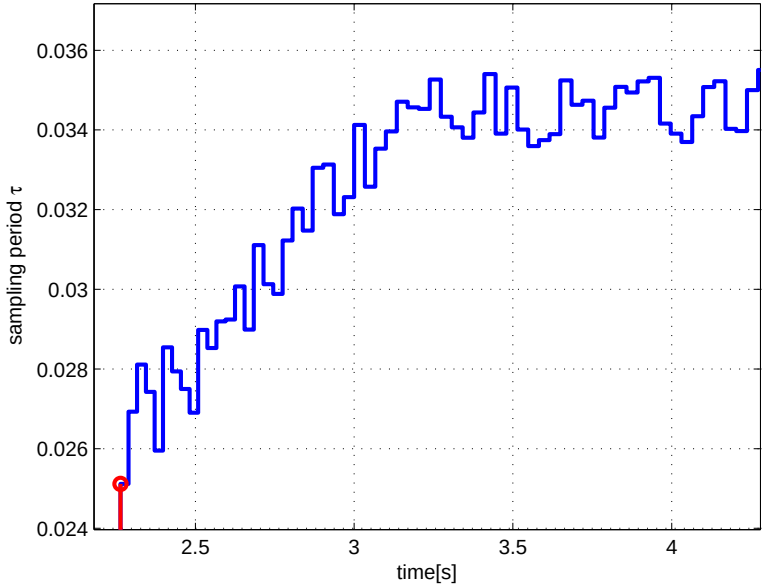


Trajectory Tracking - with Switched Modeling

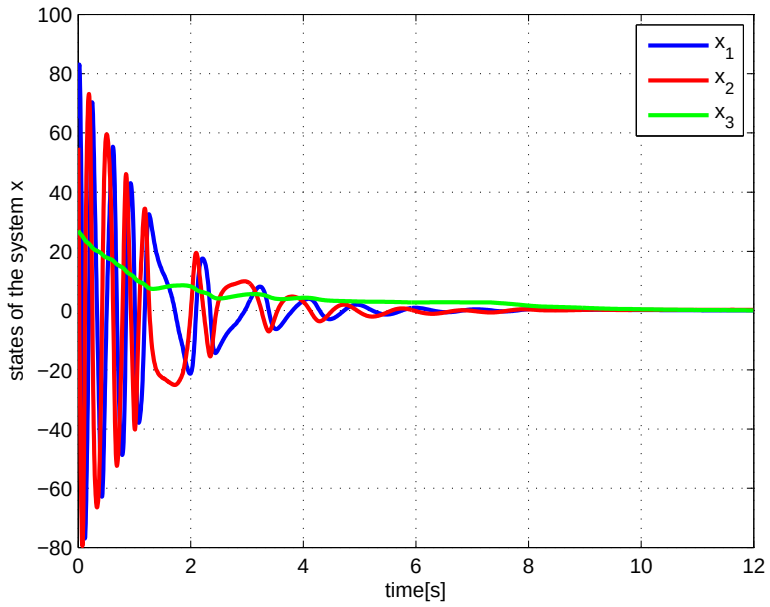


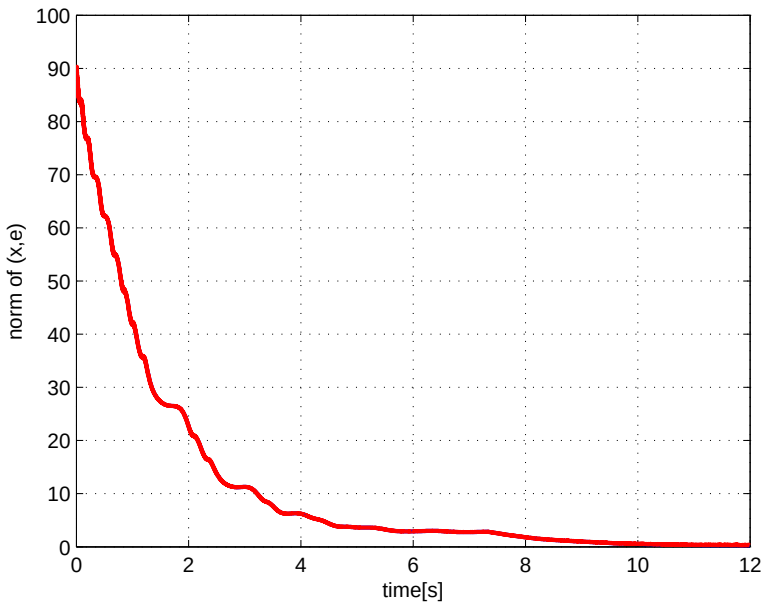


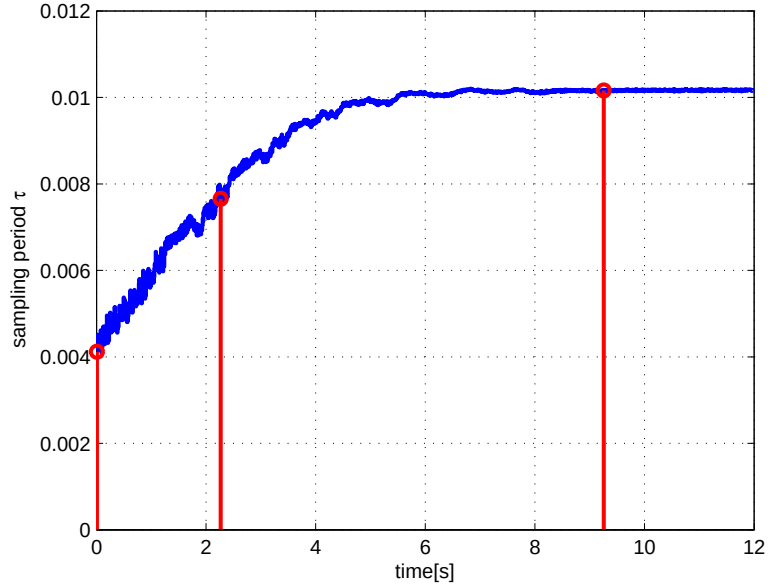




Trajectory Tracking - without Switched Modeling







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- think of an airplane driven by an autopilot system designed to follow the **shortest path** between two points
- any **deviation** from the shortest path caused by intermittent feedback **increases** total **fuel consumption**
- this increase in fuel consumption is probably **more costly** than the cost of energy saved due to intermittent feedback

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- any **deviation** from the shortest path caused by intermittent feedback **increases** total **fuel consumption**
- this increase in fuel consumption is probably **more costly** than the cost of energy saved due to intermittent feedback
- we encode these **energy consumption trade-offs** in a cost function, and design an **Approximate Dynamic Programming** (ADP) approach that yields **optimal intertransmission intervals** with respect to the cost function

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- Tolić, D.; Fierro, R.; Ferrari, S.; , "Optimal Self-Triggering for Nonlinear Systems via Approximate Dynamic Programming," 2012 IEEE Multiconference on Systems and Control (MSC 2012), 2012, accepted for publication

Performance Index - Dynamic Programming I

Goal

Minimize the following **cost function** $V : \mathbb{R}^{n_x} \to \mathbb{R}$

$$V_{\tau_i}(x_0) = \mathop{\mathbb{E}}_{e_\omega}\left\{ \sum_{i=1}^{\infty} \gamma^i \Big[\underbrace{\int_{t_{i-1}}^{t_i} (x_p^T Q x_p + u^T R u)\mathrm{d}t + S}_{l(x_p,u,\tau_i)} \Big] \right\} \tag{14}$$

over all **sampling policies** τ_i and for all initial conditions $x_0 \in \mathbb{R}^{n_x}$.

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over all **sampling policies** τ_i and for all initial conditions $x_0 \in \mathbb{R}^{n_x}$.

This cost function captures **performance vs. energy** trade-offs.

Performance Index - Dynamic Programming II

- minimization of (14) is equivalent to **Hamilton-Jacobi-Bellman equation**

$$V^*(z) = \inf_{\tau \in [0, \tau^{\max}]} \Big(l(z, u, \tau) + \gamma \mathop{\mathbb{E}}_{e_\omega} \{ V^*(f(z, u, \tau, \hat{\omega}_p, e_\omega)) \} \Big).$$

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- $V^*(z)$ is called the **optimal value function** (or optimal cost-to-go function)
- $V^*(z)$ represents the **cost** incurred by an **optimal policy** τ^* when the initial condition in (14) is z.

Performance Index - Dynamic Programming III

- introduce the **Bellman operator** $\mathcal{M}$ as

$$\mathcal{M}g = (\mathcal{M}g)(z) = \inf_{\tau \in [0, \tau^{\max}]} \left(l(z, u, \tau) + \gamma \mathop{\mathbb{E}}_{e_\omega} \{ g(f(z, u, \tau, \hat{\omega}_p, e_\omega)) \} \right)$$

for any $g : \mathbb{R}^{n_x} \to \mathbb{R}$

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for any $g : \mathbb{R}^{n_x} \to \mathbb{R}$

- since $\gamma \in (0, 1)$, therefore $\mathcal{M}$ is a **contraction**, i.e.,

$$\|\mathcal{M}u - \mathcal{M}v\|_s \leq \gamma \|u - v\|_s$$

where $\|v\|_s = \sup_{z \in \mathbb{R}^{n_p}} v(z)$

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$$\|\mathcal{M}u - \mathcal{M}v\|_s \leq \gamma \|u - v\|_s$$

  where $\|v\|_s = \sup_{z \in \mathbb{R}^{n_p}} v(z)$
- the set $\mathcal{B}$ of all bounded, real valued functions with the norm $\| \cdot \|_s$ is a **Banach space**

Performance Index - Dynamic Programming III

- introduce the **Bellman operator** $\mathcal{M}$ as

$$\mathcal{M}g = (\mathcal{M}g)(z) = \inf_{\tau \in [0, \tau^{\max}]} \left(l(z, u, \tau) + \gamma \mathop{\mathbb{E}}_{e_\omega} \{ g(f(z, u, \tau, \hat{\omega}_p, e_\omega)) \} \right)$$

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  where $\|v\|_s = \sup_{z \in \mathbb{R}^{n_p}} v(z)$
- the set $\mathcal{B}$ of all bounded, real valued functions with the norm $\| \cdot \|_s$ is a **Banach space**
- therefore, for each initial $V^0 \in \mathcal{B}$, the **sequence of value functions** $V^{n+1} = \mathcal{M}V^n = \mathcal{M}^{n+1}V^0$ converges to V^*

Issues

1. "Curses of dimensionality"

$$(\mathcal{M}g)(z) = \inf_{\tau \in [0,\tau^{\max}]} \left(l(z,u,\tau) + \gamma \mathop{\mathbb{E}}_{e_\omega} \{ g(f(z,u,\tau,\hat{\omega}_p, e_\omega)) \} \right)$$

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3. Partially observable states

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This is why we use **Approximate Dynamic Programming**.

Curses of Dimensionality

$$(\mathcal{M}g)(z) = \inf_{\tau \in [0,\tau^{\max}]} \left(l(z,u,\tau) + \gamma \mathop{\mathbb{E}}_{e_\omega} \{ g(f(z,u,\tau,\hat{\omega}_p, e_\omega)) \} \right)$$

1. Uncountable and multi-dimensional state space
   –> choose a **finite set** of points $\mathcal{X} \subset \mathcal{C}_x \subset \mathbb{R}^{n_x}$

2. Computing expectation
   –> a sum of a **quadrature approximation** (e.g., Simpson formula)

3. Optimization
   –> **gradient search** methods with constraints starting from different initial points

Approximation Architecture I

- introduce an approximate value function $\hat{V}^i$ of V^i
- compute $\hat{V}^{i+1}$ only for the points in $\mathcal{X}$
- **generalize/interpolate** for $\hat{V}^{i+1}$ for $\mathcal{C}_x \setminus \mathcal{X}$

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Properties

(i) $\hat{V}^{i+1}(x') = (\mathcal{M}\hat{V}^i)(x')$;

(ii) $\mathrm{supp}(\hat{V}^{i+1} - \hat{V}^i) = \mathcal{C}_i$, where $\mathcal{C}_i \subset \mathcal{C}_x$ is a convex and compact neighborhood of x'; and

(iii) for any $c \in \partial\mathcal{C}_i$ the following holds

$$\hat{V}^{i+1}[S] \subseteq [\hat{V}^{i+1}(c), \hat{V}^{i+1}(x')],$$

where $\hat{V}^{i+1}[S]$ is the image of the segment $\mathcal{S}$ connecting x' and c.

Approximation Architecture II

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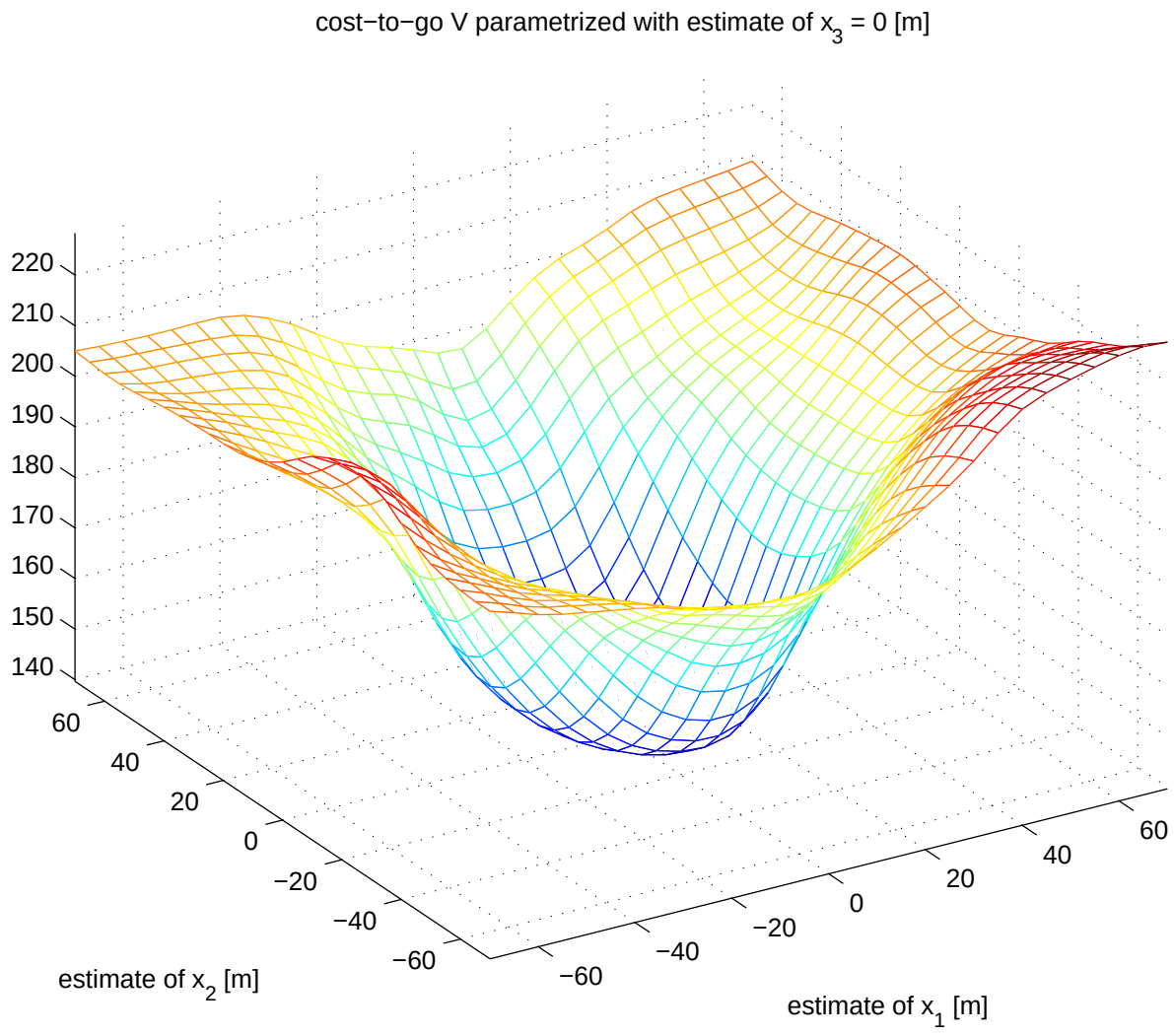
Approximation Architecture II

- **online learning** of Neural Networks (NNs)
- for **batch learning**, properties (i), (ii) and (iii) cannot be guaranteed since NNs are **expansion approximators**
- exceptions are **kernel-based** NNs and **recurrent** NNs in certain settings
- **randomly** pick points $x_i' \in \mathcal{C}_x$ in each step
  - we do not have to specify $\mathcal{X}$
  - we avoid the problem of **exploration vs. exploitation** (see [Powell, 2007] and [Sutton and Barto, 1998])

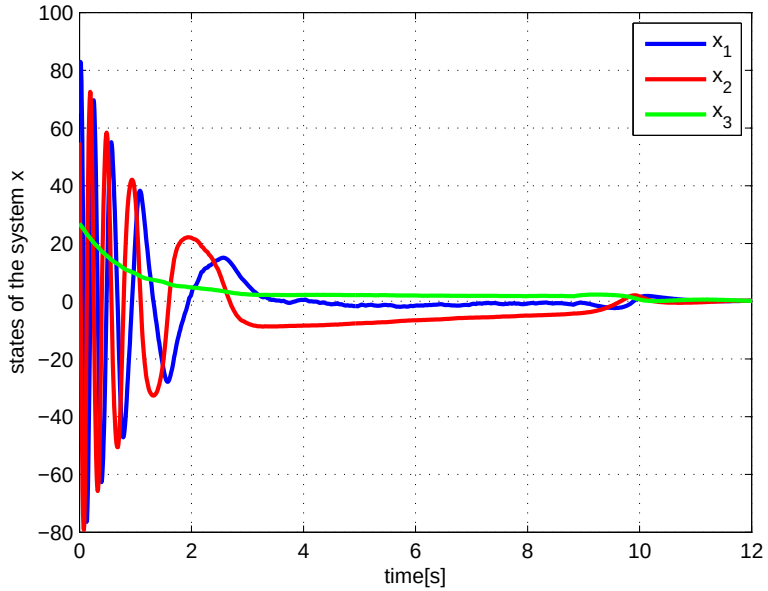
Partially Observable States

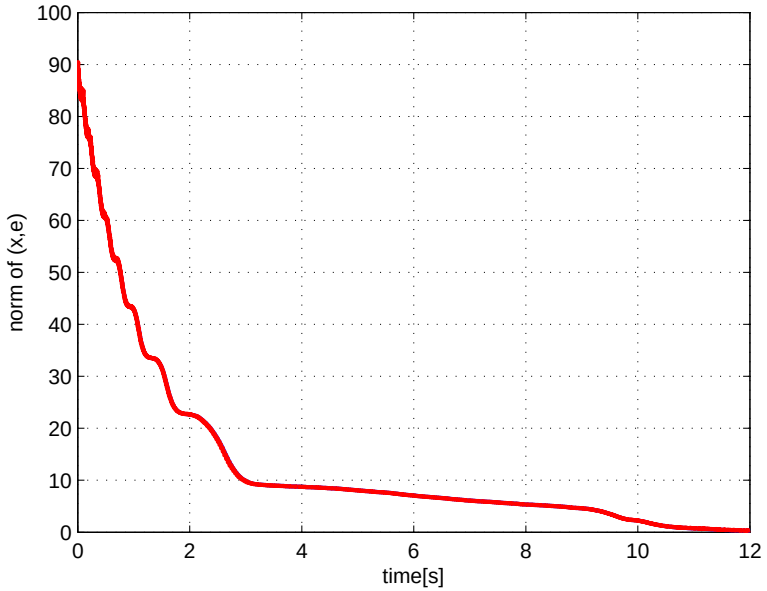
- we assume that the controller can access its state x_c
- consequently, the controller can access u at any given time
- the controller does not have access to the state of the plant x_p but merely to $\hat{\omega}_p$ and $\hat{y}$
- we propose a **particle filter** to extract $\hat{x}_p$ from $\hat{\omega}_p$ and $\hat{y}$ and feeds the controller

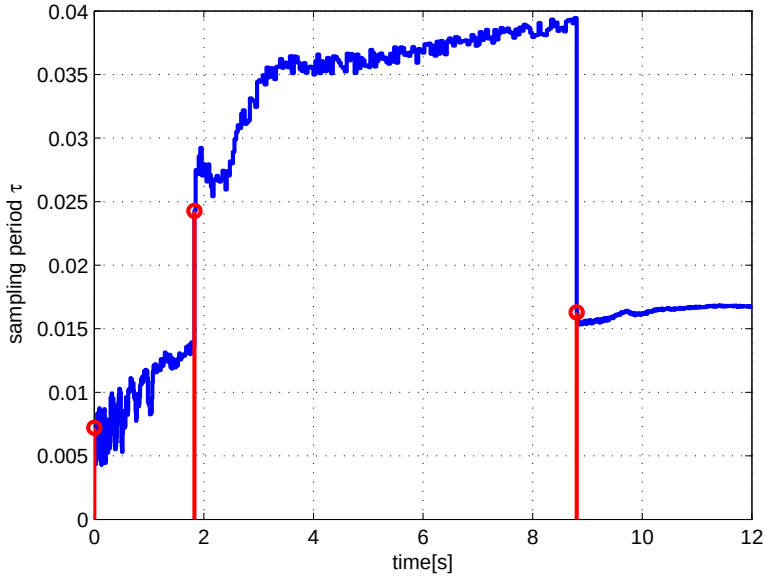
Trajectory Tracking

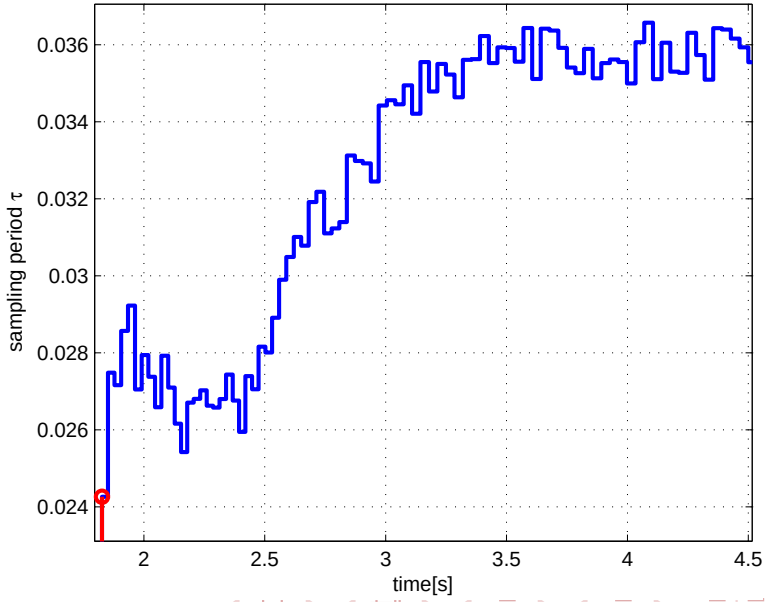


Results - Trajectory Tracking









Outline

1. Introduction
2. Stability
3. Optimal Intermittent Fdbk
4. Summary

Summary

- **input-output triggered control** for **nonlinear systems**
- the **small-gain theorem** is employed to prove stability
- $\mathcal{L}_p$-gains over a **finite horizon**
- novel results regarding $\mathcal{L}_p$-stability of **hybrid systems** are presented
- **optimal** self-triggering via **Approximate Dynamic Programming**

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Questions? Comments? Suggestions?

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Thank You for Your attention!!