Bearing-Only Tracking with a Mixture of von Mises Distributions

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ACROSS Centre of Research Excellence
for Advanced Cooperative Systems

Outline

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Motivation

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- angular random variables $\Rightarrow$ von Mises distribution
- captures well non-euclidean properties of angular data

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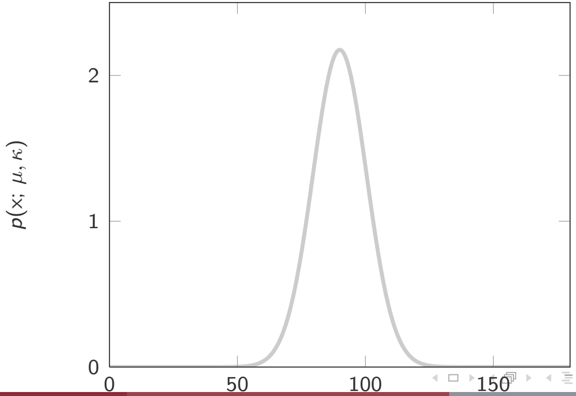
von Mises Distribution

- the pdf has the following form [von Mises, 1918]

$$p(x; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} \exp\left[\kappa \cos(x - \mu)\right],$$

where μ is the mean direction, κ is the concentration parameter, $I_0(\kappa)$ is the modified bessel function of the first kind and order zero



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Recursive Bayesian Tracking

- goal is to estimate $p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k})$

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- prediction via total probability theorem

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}) = \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})\,d\boldsymbol{x}_{k-1}$$

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- update via Bayes' rule

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k}) = \frac{p(\boldsymbol{z}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})}{p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1})}$$

Convolution

- prediction — convolution of von Mises distribution [Mardia and Jupp, 1999]

$$h(x) = \frac{1}{2\pi I_0(\kappa_i) I_0(\kappa_j)} \cdot I_0 \left(\left\{ \kappa_i^2 + \kappa_j^2 + 2\kappa_i\kappa_j + \cos(x - [\mu_i + \mu_j]) \right\}^{1/2} \right)$$

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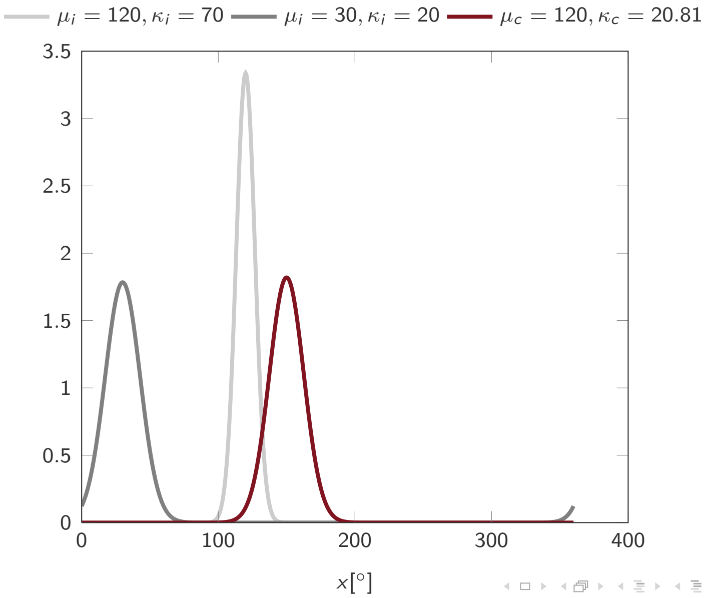
- can be well approximated by

$$h(x) \approx p(x; \mu_i + \mu_j, A^{-1}(A(\kappa_i)A(\kappa_j)),$$

where $A(\kappa) = \frac{I_1(\kappa)}{I_0(\kappa)}$

Convolution



Product

- product of von Mises distribution [Murray and Morgenstern, 2010]

$$g(x) = \frac{1}{4\pi^2 I_0(\kappa_i) I_0(\kappa_j)} \exp\left[\kappa_{ij} \cos(x - \mu_{ij})\right],$$

where

$$\mu_{ij} = \mu_i + \operatorname{atan2}\left[-\sin(\mu_i - \mu_j), \kappa_i/\kappa_j + \cos(\mu_i - \mu_j)\right],$$
$$\kappa_{ij} = \sqrt{\kappa_i^2 + \kappa_j^2 + 2\kappa_i\kappa_j \cos(\mu_i - \mu_j)},$$

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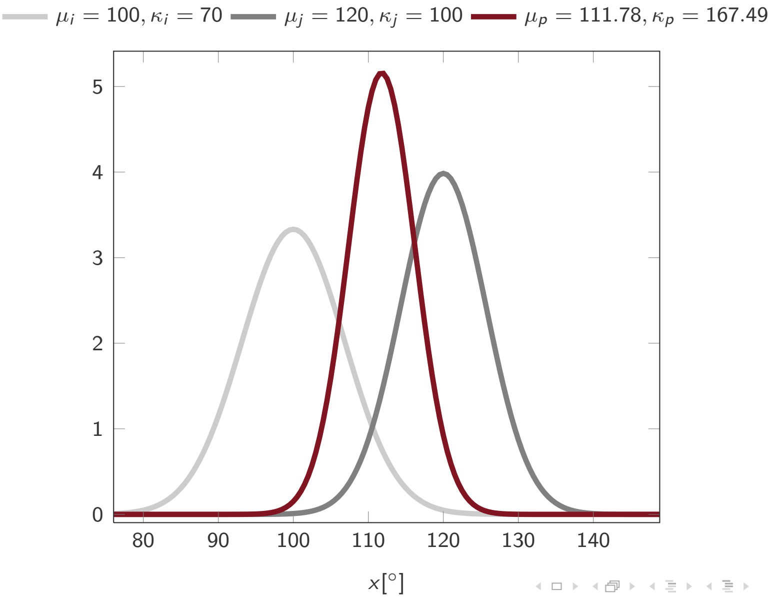
$$\kappa_{ij} = \sqrt{\kappa_i^2 + \kappa_j^2 + 2\kappa_i\kappa_j\cos(\mu_i - \mu_j)},$$

- we approximate the product with

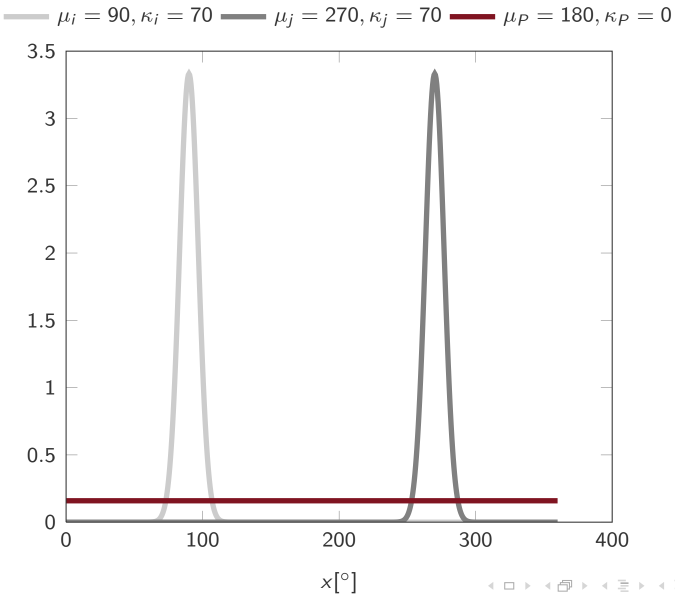
$$g(x) \approx p(x; \mu_{ij}, \kappa_{ij})$$

Product



$\mu_i = 100, \kappa_i = 70$
$\mu_j = 120, \kappa_j = 100$
$\mu_p = 111.78, \kappa_p = 167.49$
5
4
3
2
1
0
80
90
100
110
120
130
140
$x[^\circ]$

Product



$\mu_i = 90, \kappa_i = 70$
$\mu_j = 270, \kappa_j = 70$
$\mu_P = 180, \kappa_P = 0$
3.5
3
2.5
2
1.5
1
0.5
0
0
100
200
300
400
$x[^\circ]$

von Mises Mixture

- state representation is a mixture

$$p(x_k|\boldsymbol{z}_{1:k}) = \sum_{i=1}^{N} \gamma_i \frac{1}{2\pi I_0(\kappa_i)} \exp\left[\kappa_i \cos(x_k - \mu_i)\right]$$

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- motion model is a single von Mises

$$p(x_k|x_{k-1}) = \frac{1}{2\pi I_0(\kappa)} \exp\left[\kappa \cos(x_k - x_{k-1})\right]$$

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- sensor model is a mixture

$$p(\boldsymbol{z}_k|x_k) = \sum_{i=1}^{M} \gamma_i \frac{1}{2\pi I_0(\kappa_i)} \exp\left[\kappa_i \cos(x_k - z_{k,i})\right]$$

Component Reduction

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$$c_B(p, q) = \int_0^{2\pi} \sqrt{p(\xi)q(\xi)}\,\mathrm{d}\xi$$

- for von Mises pdfs closed form result [Calderara et al., 2011]

$$c_B\left(p(x; \mu_i, \kappa_i), p(x; \mu_j, \kappa_j)\right) = \frac{I_0\left(\kappa_{ij}/2\right)}{\left\{I_0(\kappa_i)I_0(\kappa_j)\right\}^{1/2}}$$

Entropy

- quadratic Rényi entropy

$$H_2(x) = -\log \int p^2(x)\,\mathrm{d}x$$

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- for von Mises mixture closed form result

$$H_2(x) = -\log \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \frac{I_0(\kappa_{ij})}{2\pi I_0(\kappa_i) I_0(\kappa_j)}$$

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Synthetic data

- two trajectories: continuous and turn-take, two filters: mixture and particle

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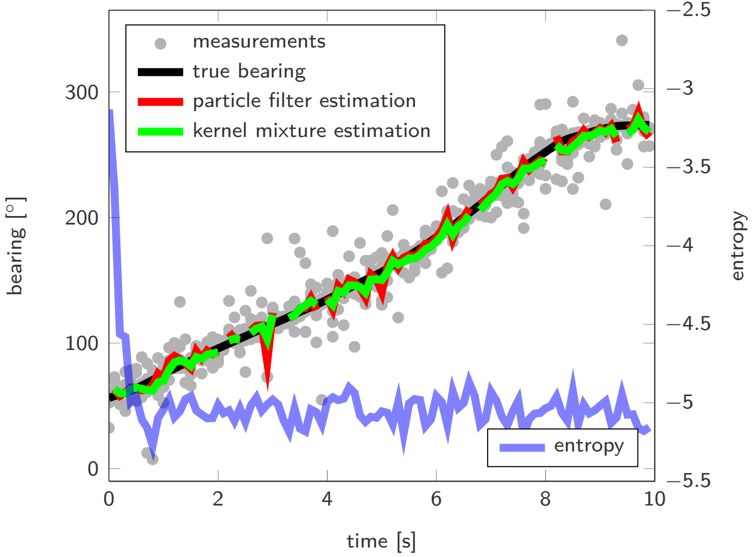
Synthetic data

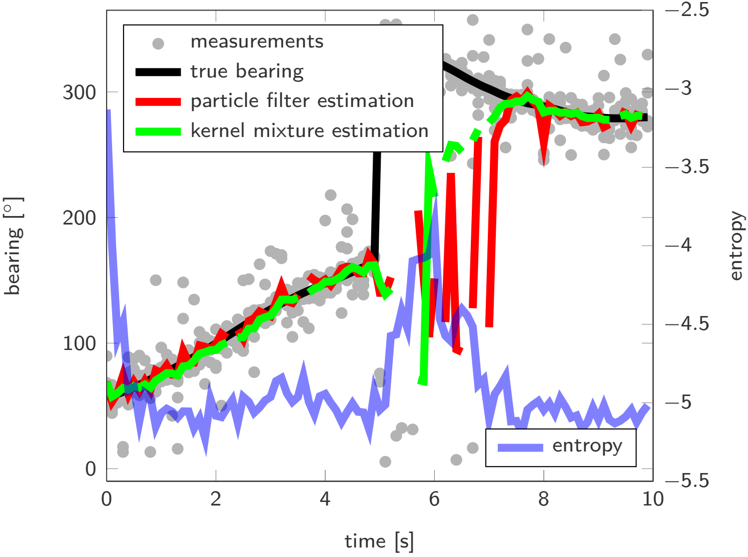
- two trajectories: continuous and turn-take, two filters: mixture and particle
- simulated multimodal measurement model (von Mises mixture) [Marković and Petrović, 2010]
- outlier probability was 0.3, while detection probability was 0.9
- measurements were corrupted with von Mises noise

Synthetic data (continuous)



Video (continuous)

Synthetic data (turn-take)



Video (turn-take)

Real-world experiments

- four omnidirectional microphones in a Y configuration

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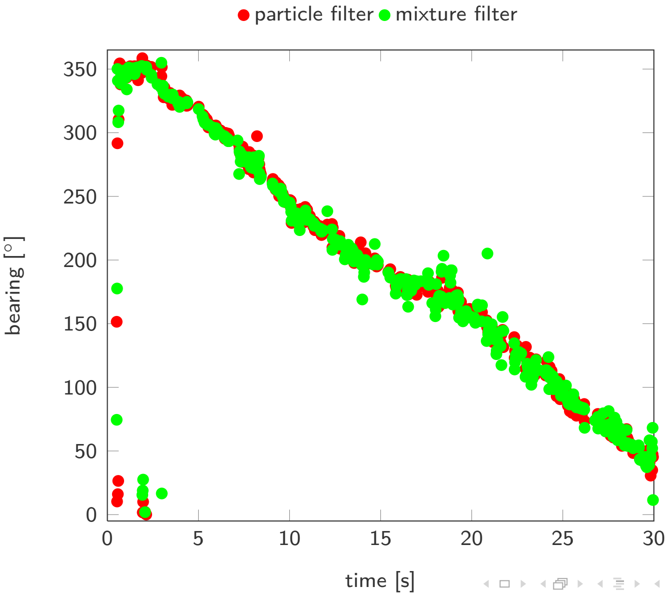
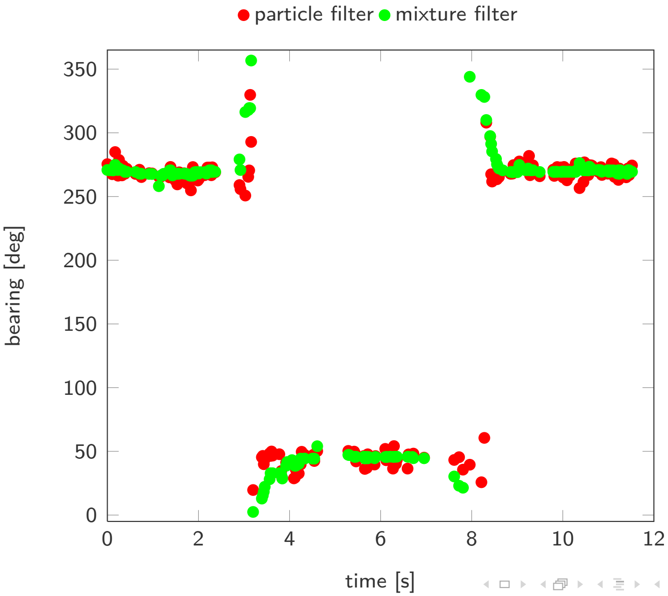
- $F_s = 48\,\mathrm{kHz}$, $L = 1024$, frame rate of approx. $47\,\mathrm{Hz}$
- two experiments: continuous from $0^\circ - 360^\circ$ and turn-take

Real-world experiments (0–360)



Real-world experiments (turn-take)



Discussion

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- 36 vs. 360 parameters for state representation
- mean time of an iteration was 81.2 ms and 72.5 ms for kernel and regularized particle filter, respectively

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- theoretical steps of Bayesian estimation with von Mises mixture
- convolution, product, component reduction and entropy
- demonstrated on, but not limited to, the problem of speaker tracking with a microphone array

Thank you for your attention

Questions?

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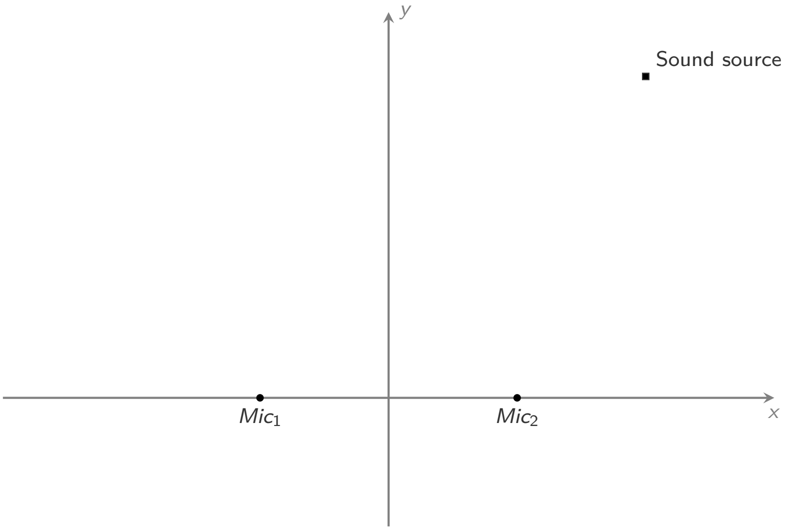
Calculating azimuth



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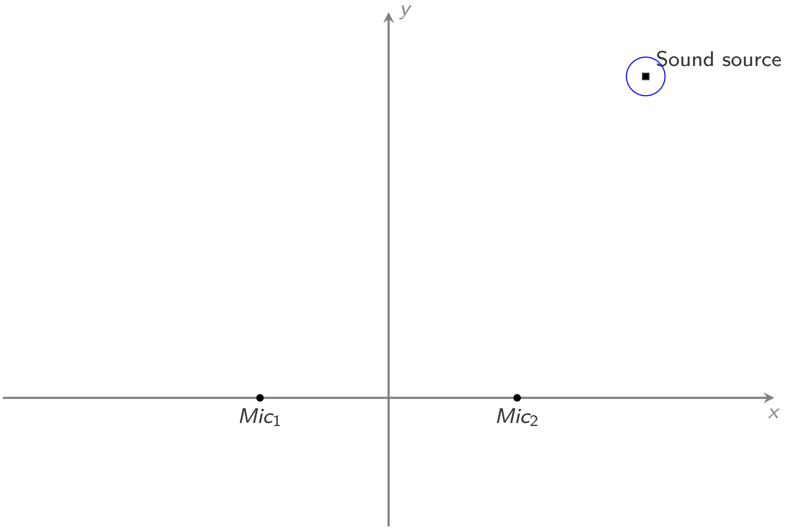

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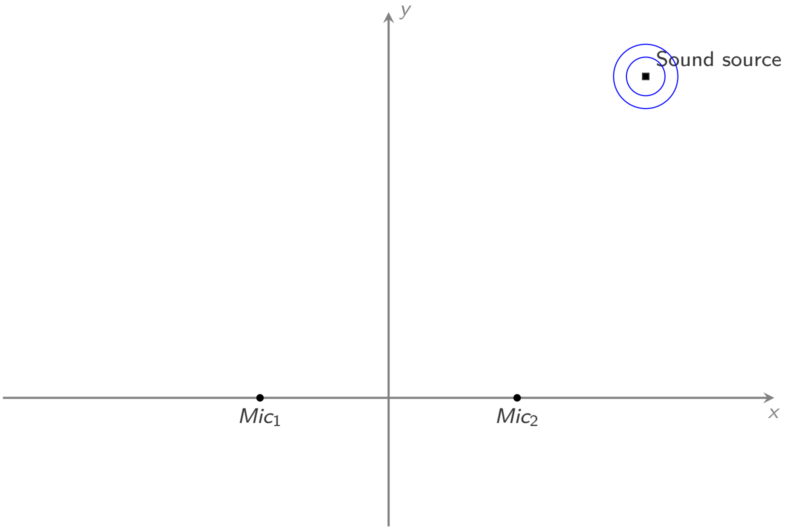

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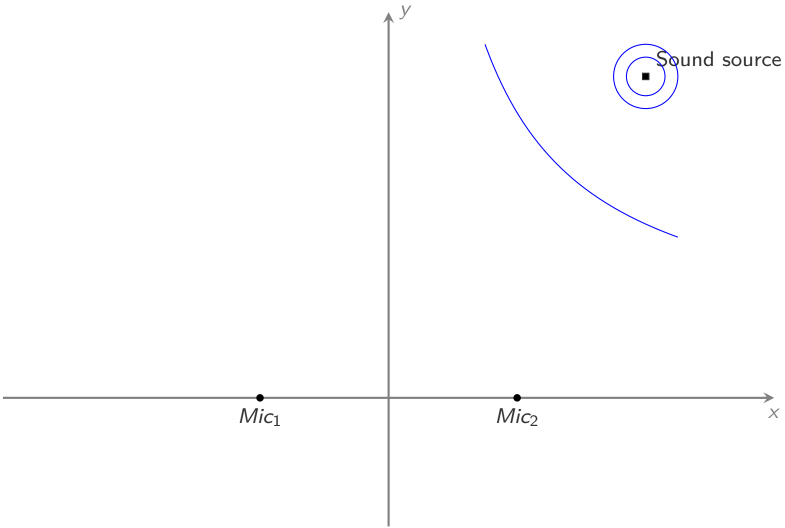

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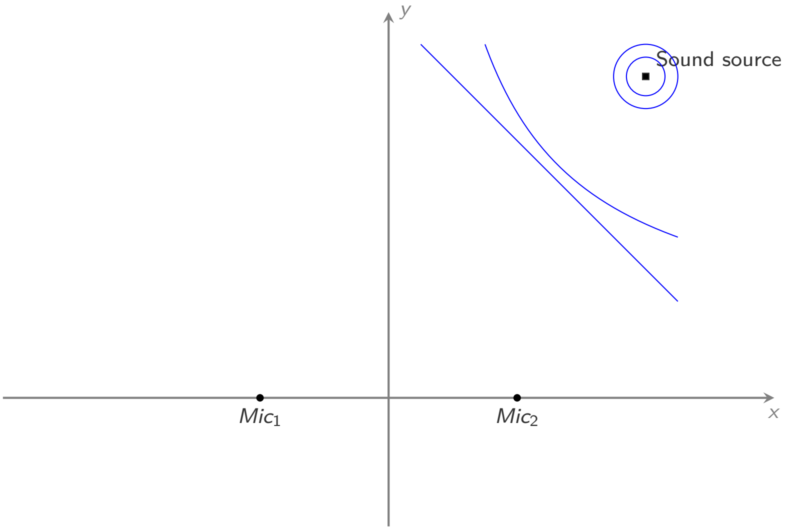

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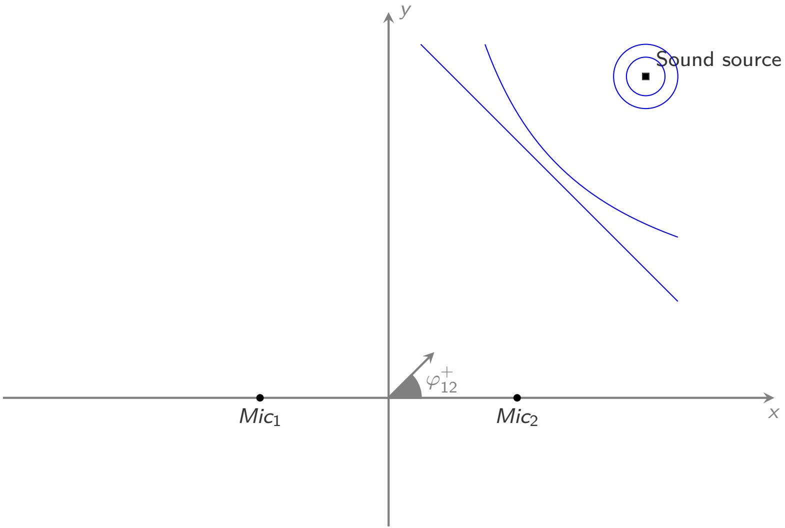

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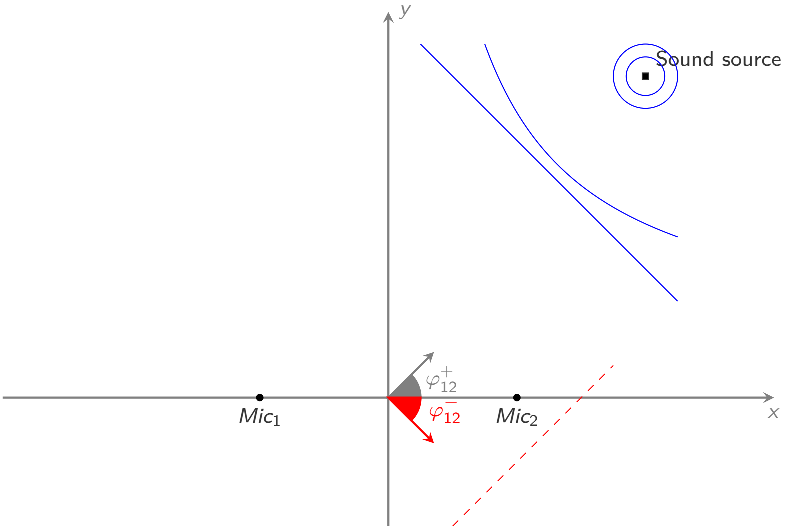

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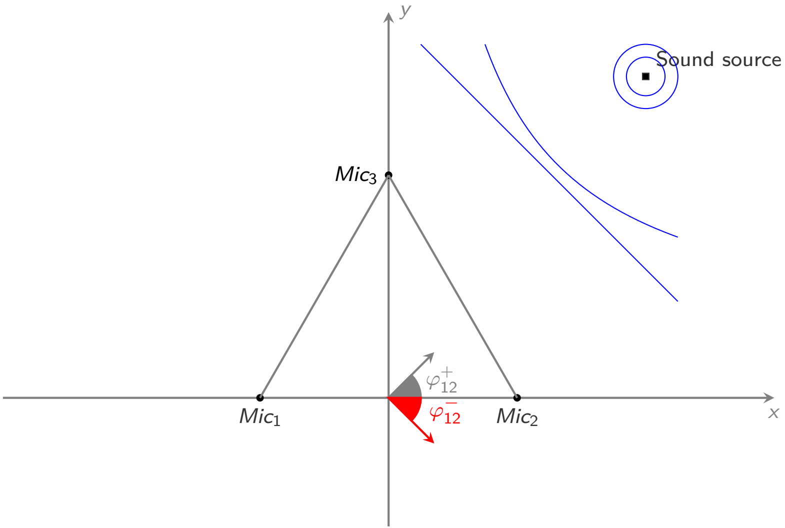

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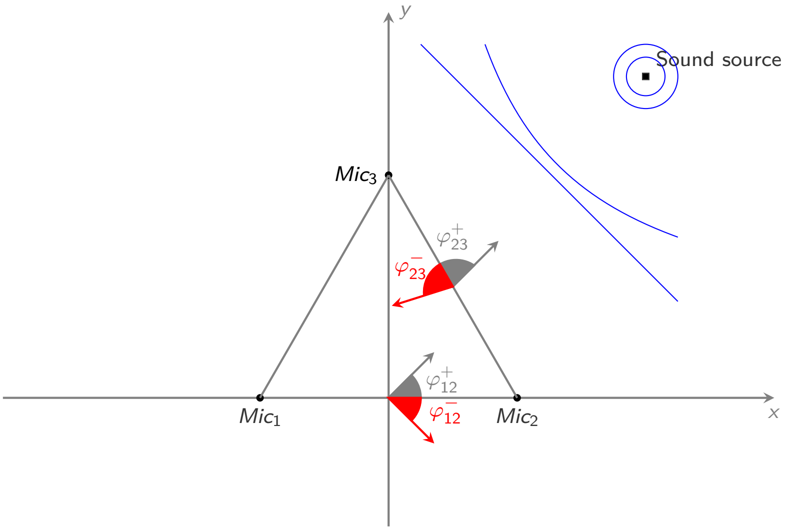

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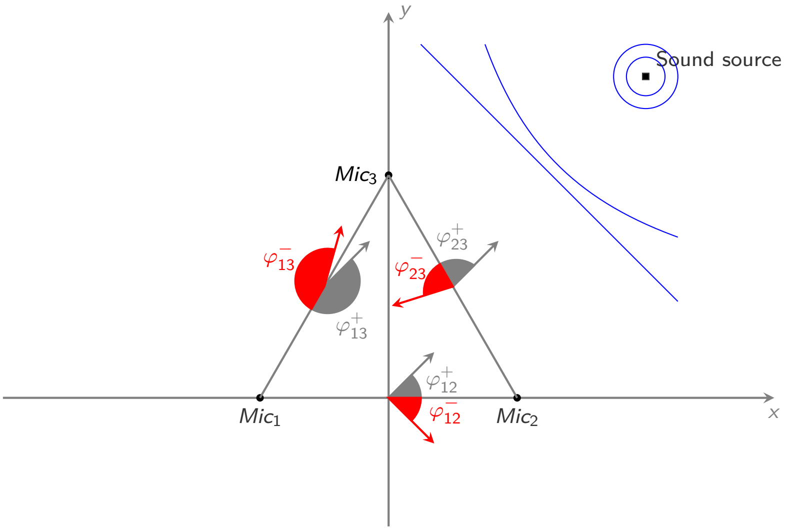
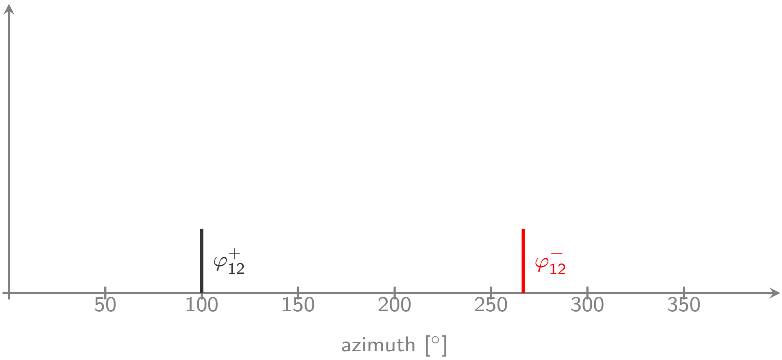


Sensor model



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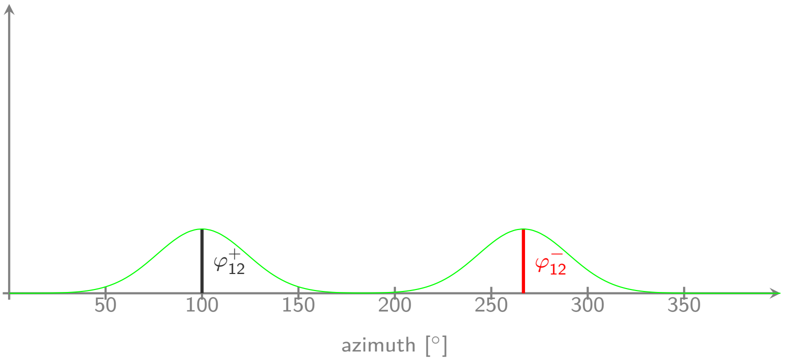

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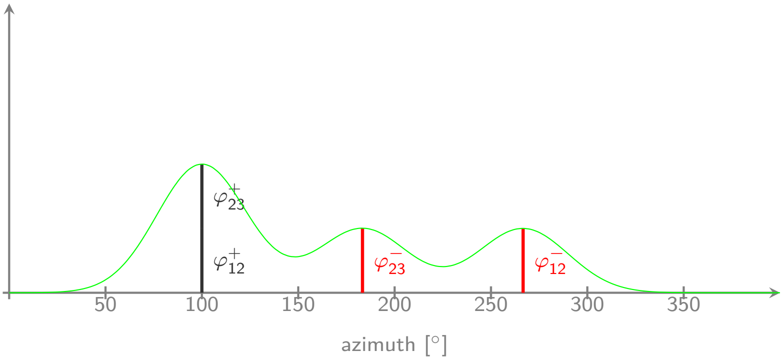

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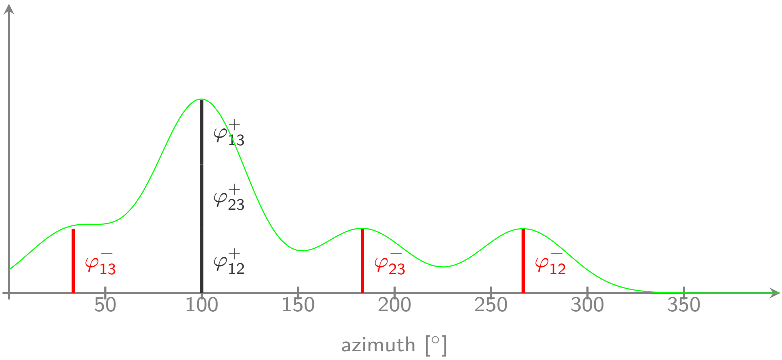

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