Design of CIC Compensators With SPT Coefficients Based on Interval Analysis

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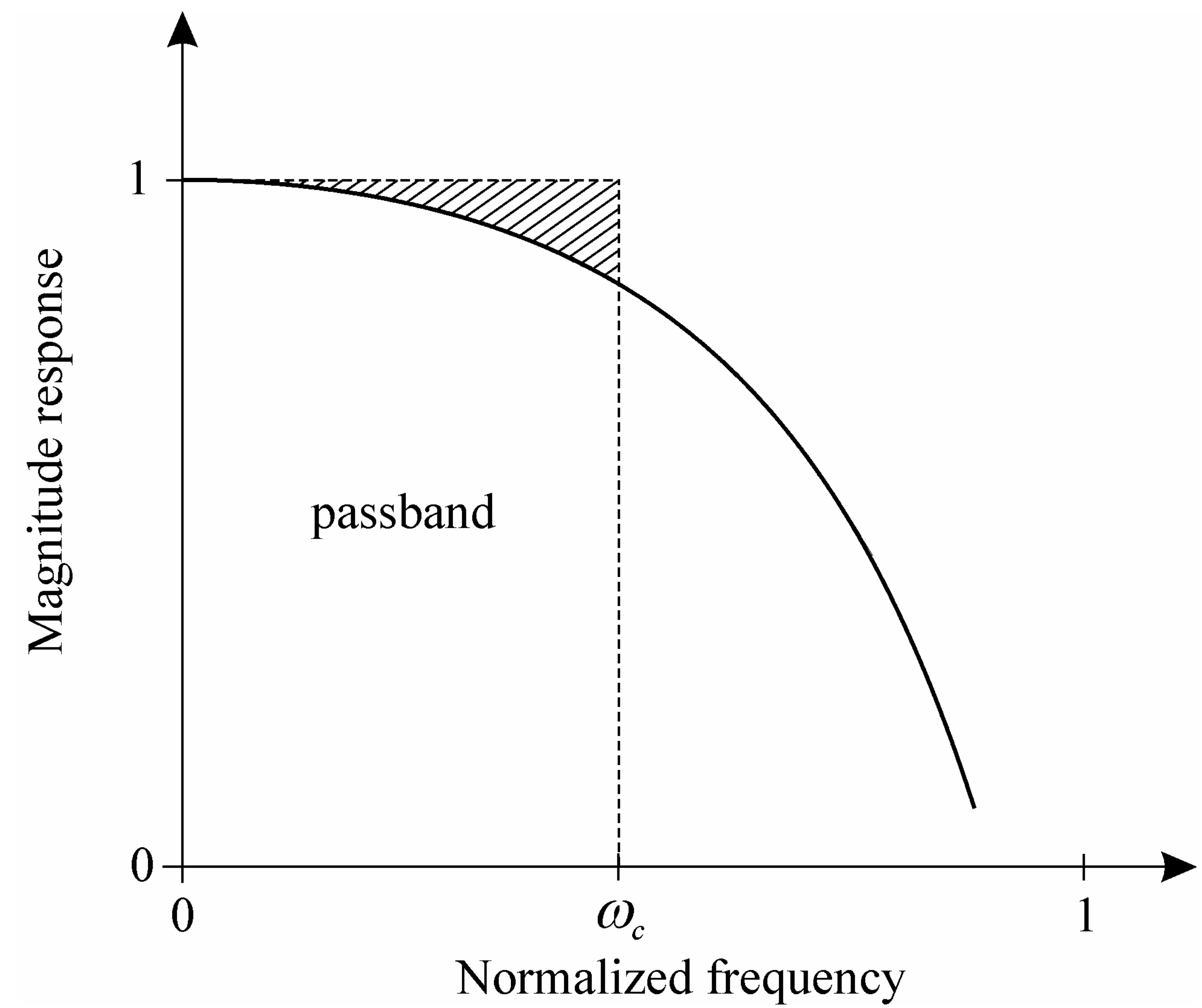
Outline

- Introduction
- CIC compensator with SPT coefficients based on minimax error criterion
- Features of proposed CIC compensators
- Conclusion

Introduction

- Digital down converters usually employ Cascaded-Integrator-Comb (CIC) filter in the first stage
- CIC filter
  - multiplierless structure
  - significant **passband droop**



Reduction of Passband Droop

- Modifying the original CIC structure
  - sharpening technique
    - Kwentus *et.al.* 1997, Stephen *et.al.* 2004, Dolecek *et.al.* 2005

- Connecting an additional filter in the cascade with the CIC decimator
  - **CIC compensator**
    - Yeung *et.al.* 2004, Kim *et.al.* 2006, Dolecek *et.al.* 2008, Dolecek 2009, Dolecek *et.al.* 2010, Molnar *et.al.* 2011, Vazquez *et.al.* 2012.

CIC Compensator

- FIR filter with linear phase
- Multiplierless structure is preferable
  - Coefficients are expressed as the sum of powers of two (SPT)
- Various multiplierless compensators have been proposed
  - Compensator with three and five coefficients
  - Compensation over wide and narrow band
- Various criteria have been used
  - Least squares, minimax, maximally flat

In this paper...

- CIC compensator with SPT coefficients
- Minimax error criterion
- Optimization based on the interval analysis
  - Results in global solution!
  - Recently, it has been used in the design of low-order FIR filters with SPT coefficients
  - Here, we modify it for the design of CIC compensator

Ideal vs. FIR Compensator

- The amplitude response of the CIC filter of order N and decimation factor R

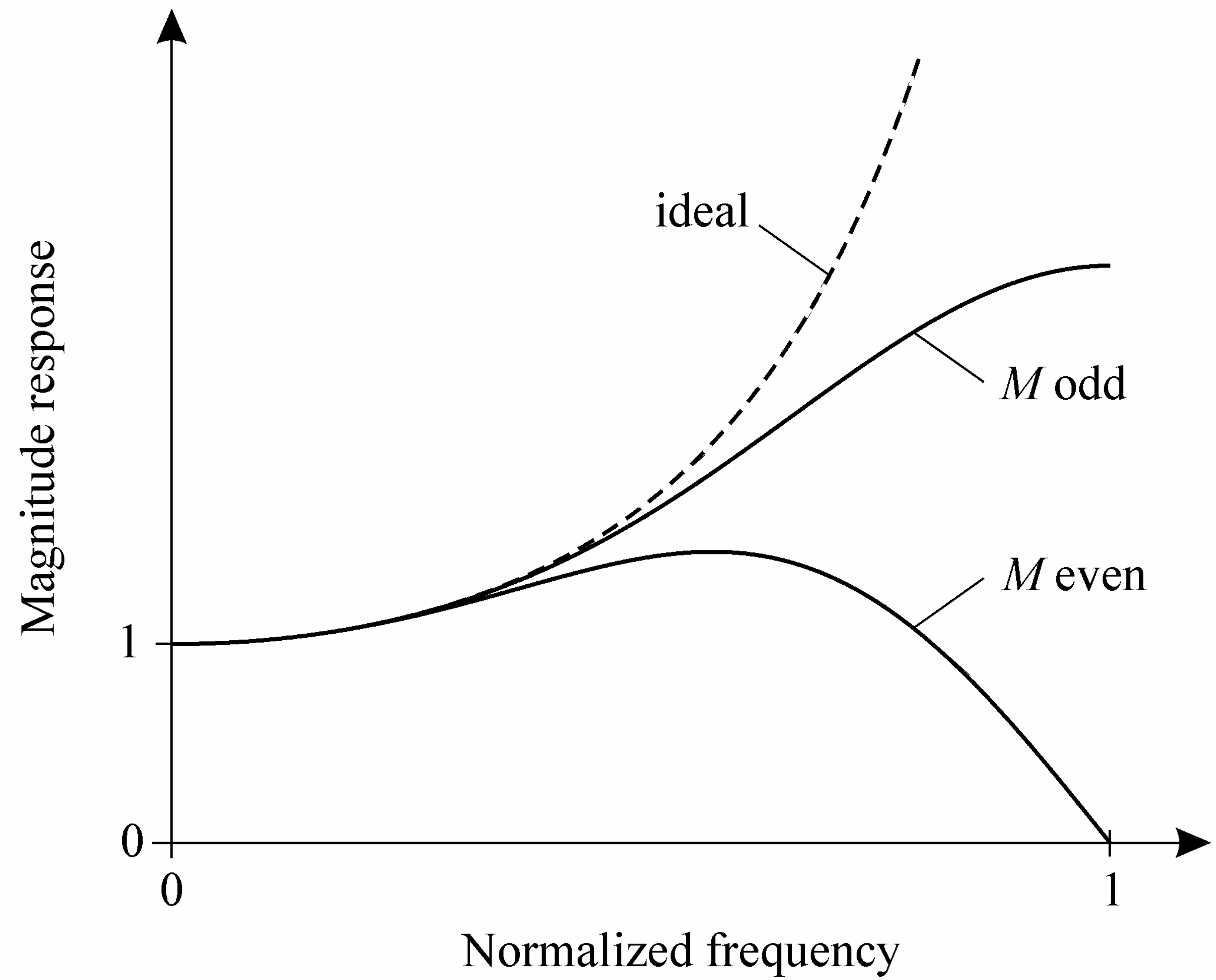
$$H_{CIC}(\omega) = \left[\frac{1}{R} \frac{\sin\left(\frac{\omega R}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} \right]^N$$

- The amplitude response of the **ideal compensator** is

$$H_C(\omega) = \frac{1}{H_{CIC}\left(\frac{\omega}{R}\right)}$$

Ideal vs. FIR Compensator

- **FIR compensator** with *M* coefficients
- Compensator with odd number of coefficients is considered



Minimax Approximation with SPT Coefficients

- Objective is to find the optimum SPT coefficients of the compensator which results in the minimax approximation

- Such a design is described by

$$\mathbf{a}_{opt} = \underset{\mathbf{a}}{\arg\min} \left[\varepsilon(\mathbf{a})\right]$$
$$\text{subject to: } \mathbf{a} \text{ is SPT representable}$$

- The **objective function** has the form

$$\varepsilon(\mathbf{a}) = \max_{|\omega| \le \omega_c} \left| 1 - H_{CIC}\left(\frac{\omega}{R}\right) H(\omega, \mathbf{a}) \right|$$

Minimax Approximation with SPT Coefficients

- The problem is solved by using the optimization based on **interval analysis**

- In the paper

  M. Vucic, G. Molnar, and T. Zgaljic, "Design of FIR filters based on interval analysis," in Proc. MIPRO, vol. MEET, 2010.

  the authors deal with the objective function

$$y(\mathbf{x}) = \max_{\omega \in \Omega} \left[W(\omega) \left| H_a(\omega, \mathbf{x}) - H_d(\omega) \right| \right]$$

- Here, the interval extension of $\varepsilon(\mathbf{a})$ can be easily obtained by using the extensions of elementary operations

Minimax Approximation with SPT Coefficients

- Vector **a** contains only right-hand side samples

$$a(m) = h\left(m + \frac{M-1}{2}\right) \quad ; \quad m = 0, 1, \ldots, \frac{M-1}{2}$$

- Amplitude response of compensator

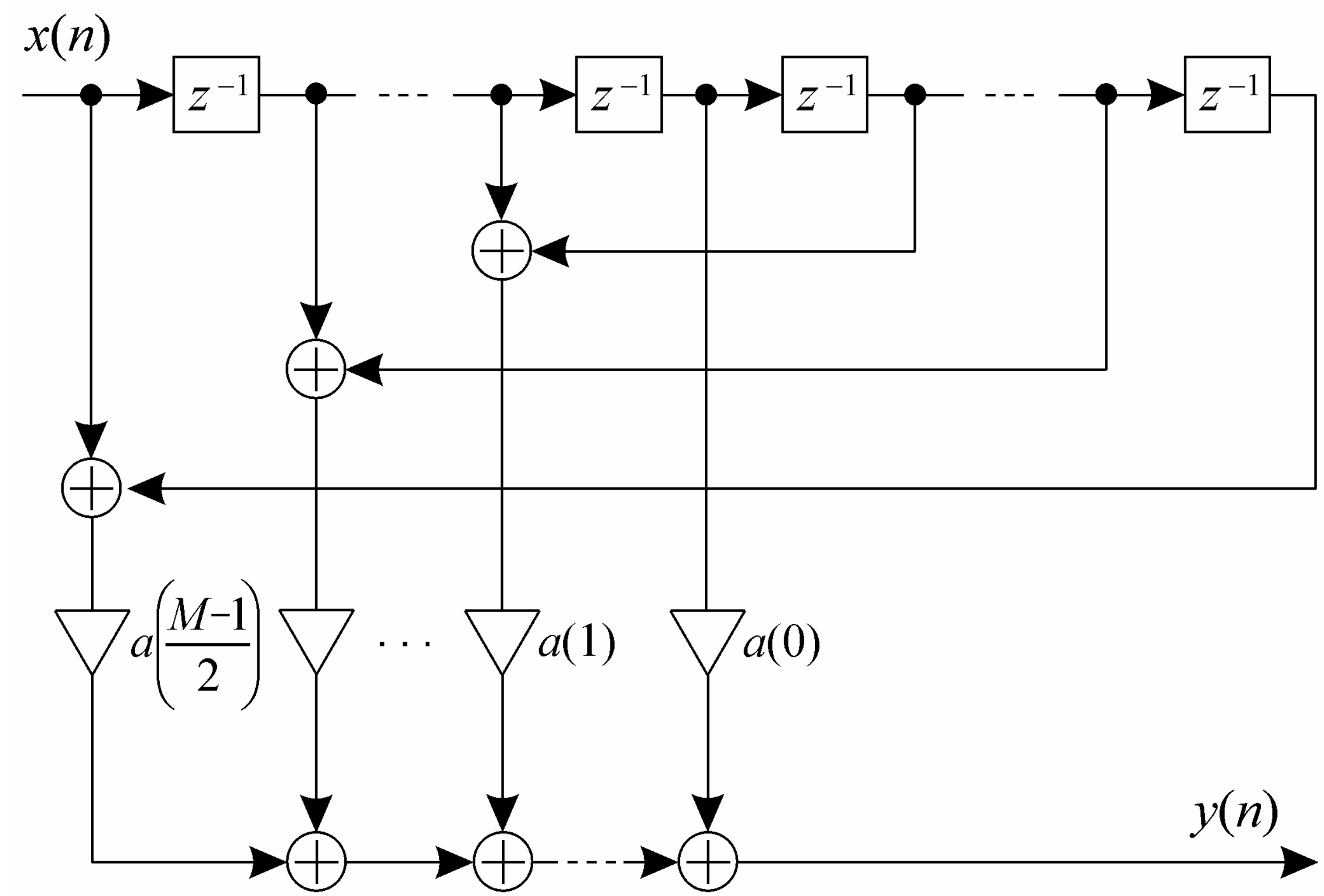
$$H(\omega, \mathbf{a}) = a(0) + 2\sum_{m=1}^{(M-1)/2} a(m)\cos(m\omega)$$

- SPT coefficient with a given wordlength K

$$a(m) = \sum_{k=1}^{K} b(m,k)2^{-k} \qquad b(m,k) \in \{-1, 0, 1\}$$

Minimax Approximation with SPT Coefficients

- Each $b(m,k)\neq 0$ represents **one adder** in hardware
- Number of terms per each coefficient is limited to a prescribed value P
- Structure of compensator



- **Total number of adders**

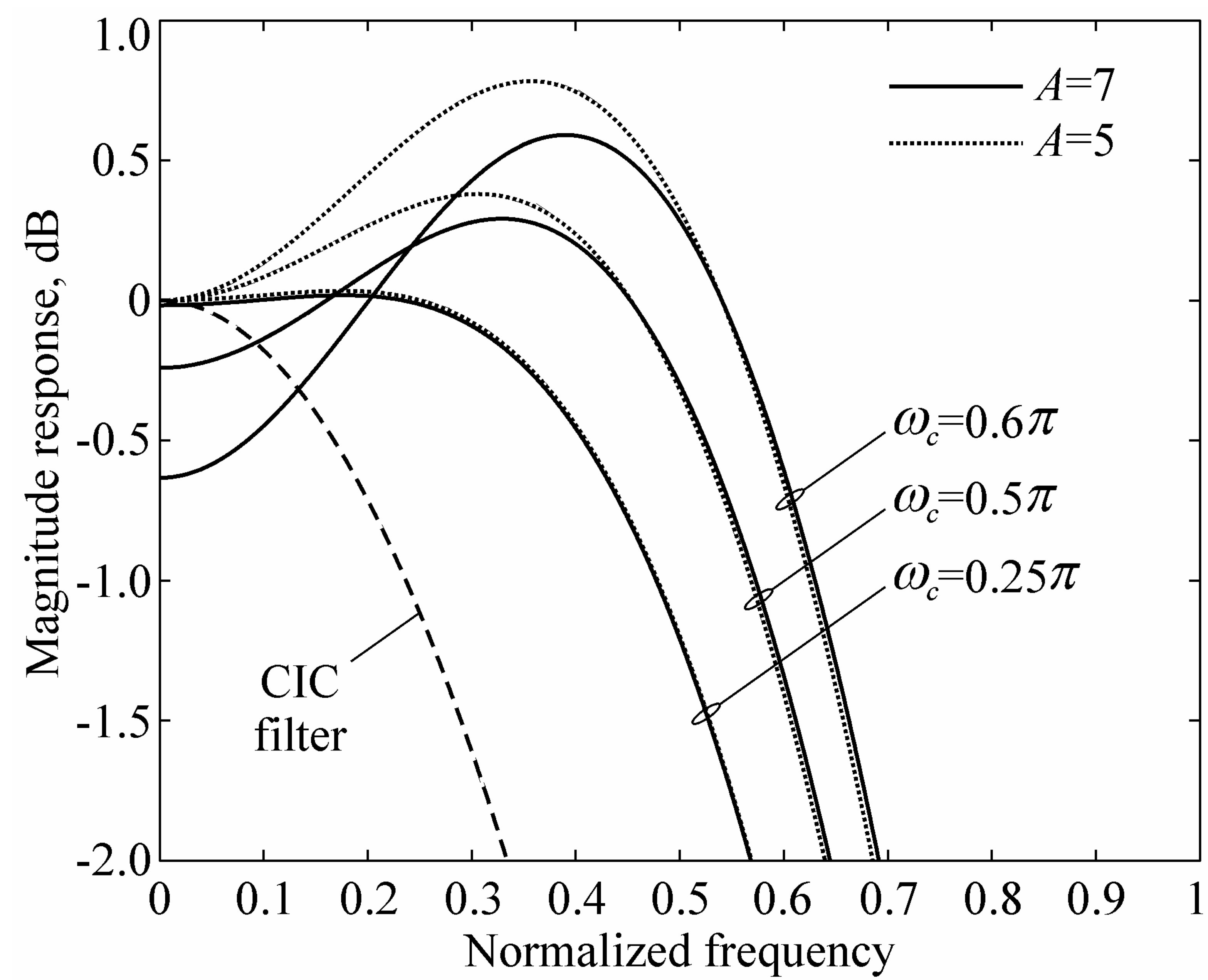
$$A \leq \frac{(M+1)(P+1)}{2} - 1$$

Features of Proposed Compensators

- Three examples are described
  - Compensators with three coefficients
  - Wideband compensators
  - Compensators with the lowest complexity

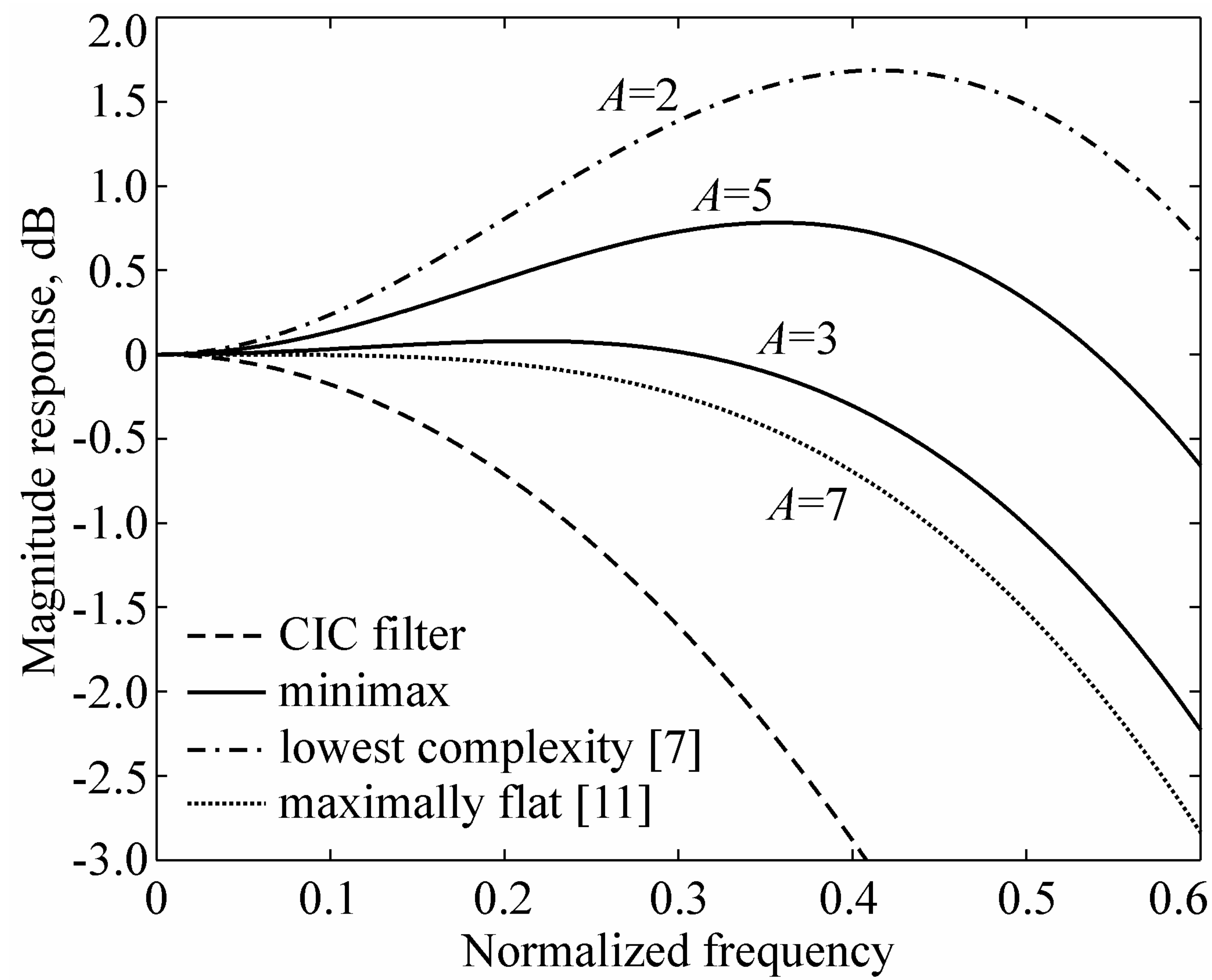
Compensators With Three Coefficients

- N=5, R=14
- Narrowband
  - ω_c=0.25π
- Wideband
  - ω_c=0.5π, 0.6π
- Two compensators
  - K=9
  - P=2 results in A=5
  - P=3 results in A=7



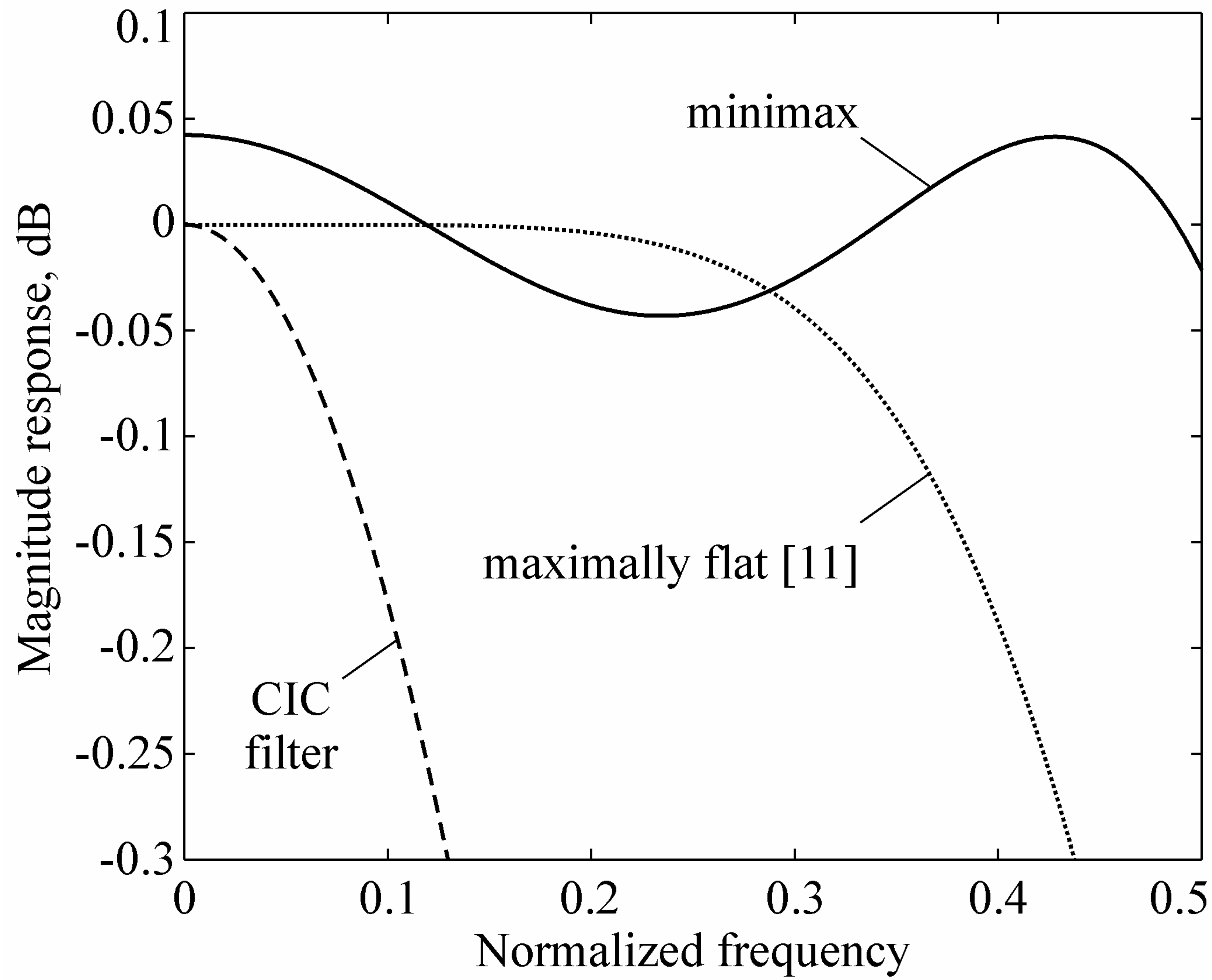
Compensators With Three Coefficients

- Comparison with other compensators
- N=5, R=14
- ω_c=0.6π



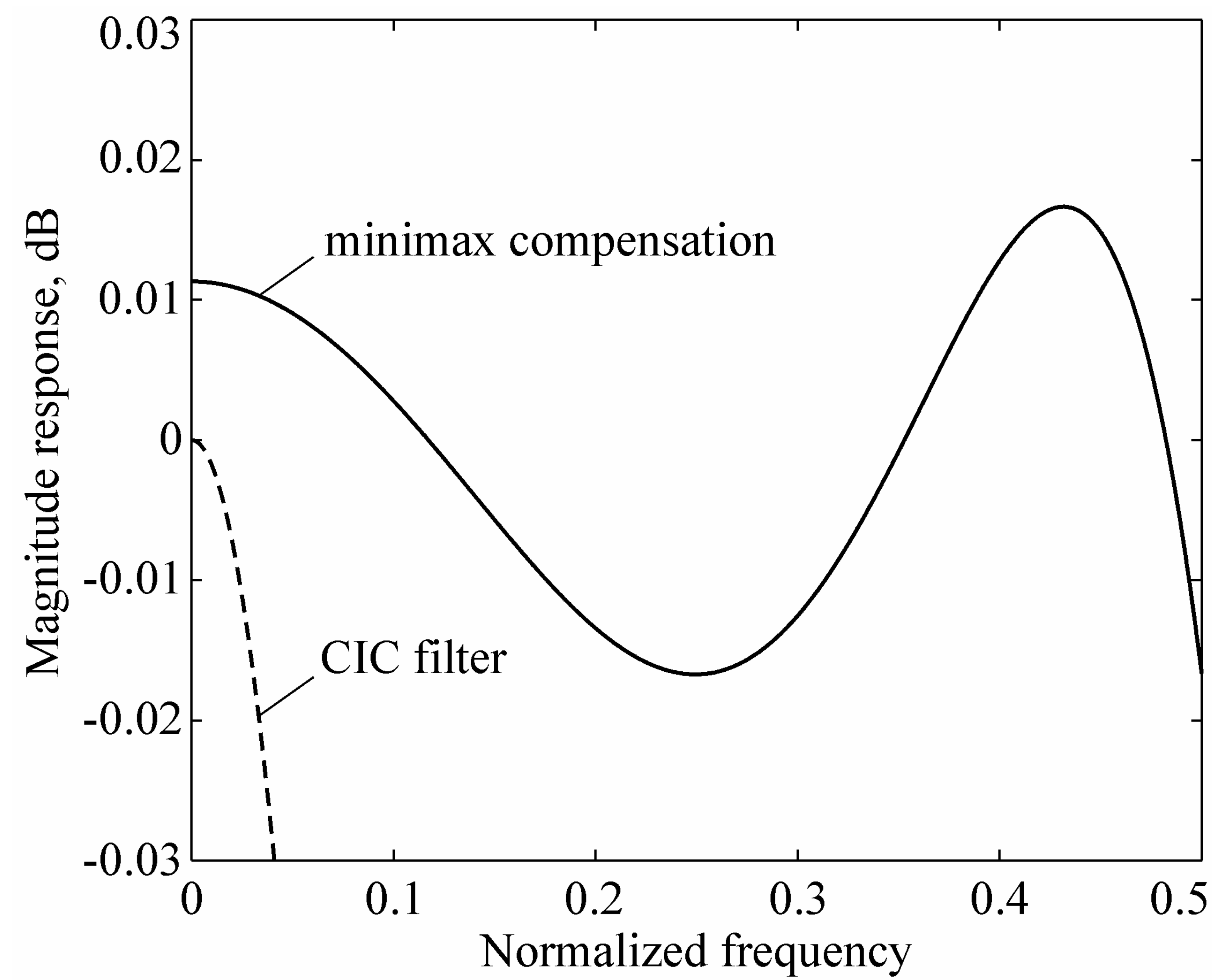
Wideband Compensators

- ***M*=5**
- N=5, R=32
- ω_c=0.5π
- MaxFlat compensator uses 15 adders
- Our compensator uses 14 adders



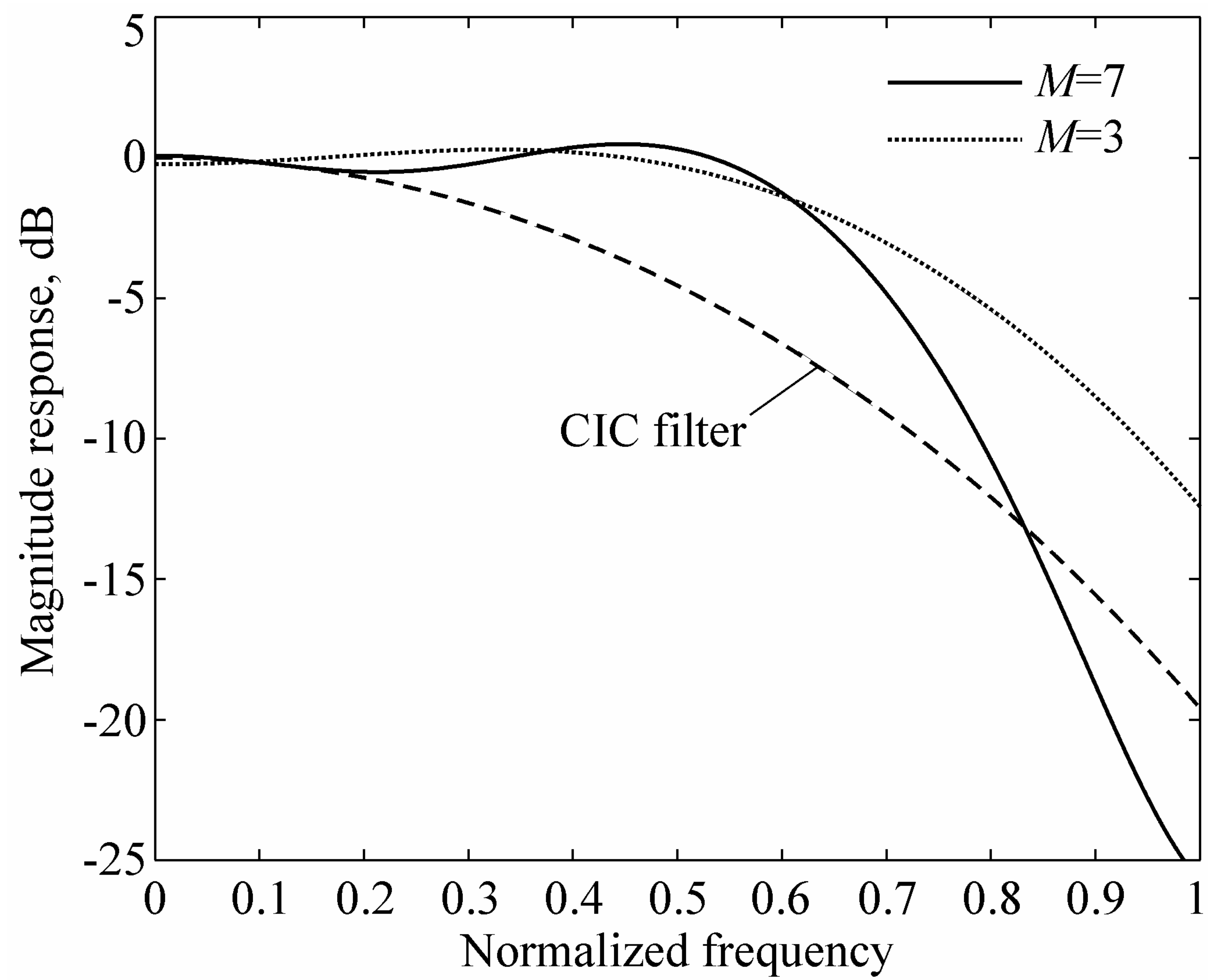
Wideband Compensators

- M=7
- Our compensator uses 19 adders
- Compensation error less than 0.02dB



Compensators With Lowest Complexity

- ***P*=1**
- N=5, R=32, ω_c=0.5π
- Two compensators
  - M=7 with P=1
  - M=3 with the same number of adders
- M=7 has a better rolloff characteristic



Conclusion

- The design of minimax CIC compensators over the **SPT coefficient space** has been presented
- The optimum compensators are obtained using the **global optimization** based on the interval analysis
  - It enables the design of the compensators with **high number of coefficients** and the **wideband** compensators
- Compared with the known SPT compensators, the proposed compensators generally result in a **better compensation** and a **lower complexity**