Quasi-stationary optimal control for wind farm with closely spaced turbines

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***Abstract*—Modern wind farms typically comprise a large number of wind turbines spread on a relatively small area. The closely spaced turbines are aerodynamically coupled since they share the common energy resource. The aerodynamical coupling is described by the wake model, which explains how the change in the energy extraction of a single wind turbine affects the operating conditions of other turbines. Wake models provide the basis for optimization of wind farm operation. The paper shows that at wind speeds lower than rated the wind farm power production can be increased by optimizing the rotor speed of individual wind turbines. Furthermore, at wind speeds above rated the power production can be redistributed among the turbines in the wind farm in order to equalize the loads and thus prevent excessive wear and tear of the exposed turbines.**

I. INTRODUCTION

The increase in world population and life standard requires corresponding increase in energy production. Fossil fuel power plants still hold the main part in energy production. Due to their negative environmental impact and limited fossil fuel reserves, much effort is invested in research of renewable energy sources, like wind energy. The nominal power, efficiency and dimensions of wind turbines have been greatly increased during last few decades. Today, the wind turbines are rarely installed as stand-alone power sources, more commonly a larger number of turbines is grouped into the wind power plant, i.e. the wind farm.

The wind farms today are typically composed of a large number of wind turbines on a relatively small area, which causes the turbines to share the common energy resource. The wind turbines extract a part of energy from the wind flow which causes a decrease in wind speed behind them. This speed deficit behind wind turbines is also called wake. Wakes cause aerodynamical coupling between wind turbines in wind farm. Due to wakes, every change in level of power extraction of a single wind turbine affects the turbines located downwind.

Traditionally, wind farm is operated as a collection of individually controlled wind turbines [8], which is not necessarily production-wise the best operating strategy for a large number of closely-spaced turbines. When a wind turbine is individually controlled it operates at its locally optimal operating point, which assures it extracts the maximum available power from the wind at its rotor. If one observes a row of turbines aligned with the wind, this control strategy allows the first upstream turbine to extract the power from the undisturbed wind flow. The wind turbines behind it experience the smaller wind speed since they are situated in the wake of the first turbine and therefore they produce less power then the first turbine. The idea we investigate in this paper is: if the power extraction of the upstream wind turbines is decreased, would the cumulative production of the turbines in the row be larger? In this paper we try to achieve such increase by model based optimal wind farm control relying on wake models.

Another opportunity for the wake model based optimization investigated in the paper arises in cases when the wind farm is required to track the externally provided power reference. This operating scenario is becoming increasingly common in the grids with large penetration of wind energy where wind energy is present at the regulating market, see e.g. [9]. Operating below the rated power under high winds provides opportunity for reallocation of wind turbine loads. With classical wind farm control the wind turbines in the front of the row produce the most power and therefore suffer the highest loading and wear. The optimization of power production based on the wake model can be used to reallocate production and loading and thus increase the operating life of the wind turbines that typically suffer the most damage.

The paper is structured as follows:
Section II describes the wind turbine and wind farm models used in the paper. Section III discusses the possibility for wind farm power optimization at lower wind speeds. In Section IV the idea of load optimization/reallocation is presented. Section V concludes the paper.

II. WIND FARM MODEL

In this paper we consider the wind farm that consists of eight wind turbines aligned in row parallel to the prevailing wind direction, see Fig. 1. The distance between the wind turbines is 400 meters. This setup is chosen because in this scenario the aerodynamical coupling between individual turbines is the strongest.

To obtain the wind farm model for wind farm optimization we require the model that inputs the mean speed of the wind flow and the control variables for each of the wind turbines. The wind farm model outputs are the power production and loads at every wind turbine. To obtain such a model we need to combine the model of the wind turbine, which links the wind turbine inputs to its production and loading, and the wake model, which links the operating conditions on one turbine to operating conditions on the turbines downwind from it.

Fig. 1. Wind farm layout

A. *Wind turbine model*

We use the NREL offshore 5 MW baseline wind turbine model developed at National Renewable Energy Laboratory, [3]. It is a model of 5 MW conventional horizontal axis three-bladed upwind variable-speed pitch-controlled wind turbine implemented in Matlab. Diameter of turbine rotor is 126 meters and tower height is 90 meters. It can operate at wide range of wind speeds: from 3 m/s to 25 m/s, while it achieves rated power at 11.4 m/s. More technical data about the wind turbine model can be found in [3].

In this paper we look at the stationary wind turbine model. The core of this model is the wind turbine aerodynamics, that describes the power capture and the aerodynamic forces that act on the wind turbine rotor due to wind stream.

The power captured by the wind turbine, P, is modeled according to the relation:

$$P = C_{\mathrm{P}} \cdot \frac{1}{2}\rho A v^3, \tag{1}$$

where C_{P} is the power coefficient, ρ is the air density, A is the rotor area and v is the velocity of the free wind stream approaching the rotor. Notice that $\frac{1}{2}\rho A v^3$ is the power available in the wind and C_{P} denotes a portion of available power captured by the wind turbine.

Similarly, the thrust force that acts on the rotor in the direction of the wind can is modeled by:

$$F_{\mathrm{T}} = C_{\mathrm{T}} \cdot \frac{1}{2}\rho A v^2, \tag{2}$$

where C_{T} is the thrust coefficient and $\frac{1}{2}\rho A v^2$ denotes the kinetic energy of the wind stream.

Both power and thrust coefficient depend on the way the turbine is operated, i.e. they depend on the blade pitch angle, β, and the blade tip speed ratio, $\lambda = \frac{\omega_{\mathrm{r}} R}{v}$, where ω_{r} denotes the rotor speed.

The control inputs of the wind turbine model are ω_{r} and β, v is an uncontrollable input, while power P and thrust force F_{T} are the outputs that represent wind turbine production and loading.

B. *Wake model*

The wake that occurs behind the wind turbine as a consequence of its energy extraction is a complex phenomenon driven by different forces and interactions. In literature different models of wakes are found depending on the downwind distance from the wind turbine on which the wake is modeled. In that context we distinguish near wake models and far wake models, see [4]. The near wake model describes the phenomena directly behind the rotor (within one rotor diameter), while the far wake models describe the phenomena further downwind. In this paper our focus is on the far wake models.

To describe the influence of wind turbine states on the far wake forming the axial induction factor a is typically used. The axial induction factor is introduced in the actuator disc theory (see [10]) to describe the wind velocity variation that occurs when the wind passes the wind turbine rotor. According to this theory the axial induction factor is related to the power as:

$$C_{\mathrm{P}} = 4a(1-a)^2, \tag{3}$$

and to the thrust coefficient as:

$$C_{\mathrm{T}} = 4a(1-a). \tag{4}$$

According to this relations the axial induction factor is a function of pitch angle and the tip speed ratio, which can be estimated from the known power and thrust coefficients.

There are several different ways to model the far wake, but most appropriate for the controller design is the engineering model found in [2]. This model contains simple expressions suitable for fast calculation. The wind speed is considered normally distributed with the mean value v and standard deviation σ, which represents the wind turbulence. At this point we are only interested in the mean wind speed.

In this paper the following expression for wind deficit is used ([2]):

$$v(x,r) = v_0 - \Delta v(x,r). \tag{5}$$

where $v(x,r)$ denotes the wind speed at distance x from the wind turbine and at the radial distance r from the wind turbine hub, v_0 is the speed of the free wind stream in front of the turbine. Δv denotes the speed deficit due to turbine power extraction, which is modeled as:

$$\Delta v(x,r) = 2 a v_0 \left(\frac{x}{2D}\right)^n e^{\dfrac{-\alpha r^2}{\gamma^2(x)}}, \tag{6}$$

where D is the rotor diameter and parameters n, α and $\gamma(x)$ are experimentally determined.

C. *Multiple wakes*

To model a wind farm for optimization one needs to compute the wind speed deficit at the position of each turbine. If one considers the wind farm layout depicted in Fig. 1 it can be noted that, for example, wind turbine denoted WT3 will be in wakes of both WT1 and WT2, while WT8 will be in wakes of all other turbines. So it is necessary to model the wake merging.

According to [2] the wind speed deficits obtained from the model (6) are additive, i.e. the influence of wakes of different turbines at a certain location should be summarized:

$$\Delta v_{\Sigma}(x,r) = \sum_{k=1}^{n} \Delta v(x_k, r_k), \tag{7}$$

where x_k denotes the distance between the point (x,r) and the k-th wind turbine along the x axis, and r_k denotes the radial distance between the point (x,r) and the center of the k-th wind turbine.

The wake effects in the considered wind farm are depicted in Figure 2. The x and y axes denote the horizontal location in the wind farm (the system origin is at the location of WT1). The vertical axes denotes the wind speed at different locations. In the figure one can notice the basic characteristics of wakes: the wind turbines cause a sharp drop in the wind speed, while the wind speed recovers further downwind.

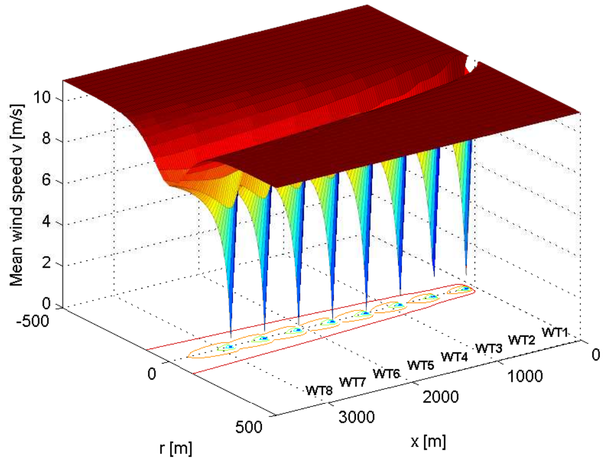

Fig. 2. Mean wind speed at wind farm area

When considering the wind farm control it is more appropriate to depict wake effects at the wind turbines. In Figures 3 and 4 the markers denote wind speed deficit and the power produced at particular wind turbines. Notice that at wind speed of 13 m/s the pitch angle is larger than zero. This is because at that wind speed the available power is larger than nominal. The produced power is then limited to nominal by blade pitching. From both figures it can be noticed that wind turbines captured in the wake produce less power. The largest drop in wind speed occurs between the first and the second wind turbine, while further on the difference is smaller. In Figure 3 it can be seen that the depth and the length of wakes depend on the mean wind speed. This is even more obvious in Figure 4 where the power produced at different figures is denoted. It can be noticed that the first turbine in a row produces much higher power than the others, which is due to the fact that the produced power is proportional to the third power of the wind speed (see (1)).

III. POWER OPTIMIZATION

The primary objective of every power plant owner is to maximize the power production because the power production directly translates to income. So we first investigate is it possible to increase the power production by using wind farm model based optimization.

The control of a single wind turbine is very well researched area, see e.g. [10], and a control approach for maximization of power production is very well established. The pitch angle is generally not used for power maximization, it is held at a value that ensures maximal power extraction, which is typically denoted as $\beta = 0°$. Further, it is observed if the power coefficient as a function of tip-speed ratio has a distinctive maximum. Keeping the operating point at that tip-speed ratio (denoted optimal tip-speed ratio, which is 7.5 for the turbine at hand) maximizes the extracted power. The optimal tip-speed ratio is maintained by adapting the generator torque.

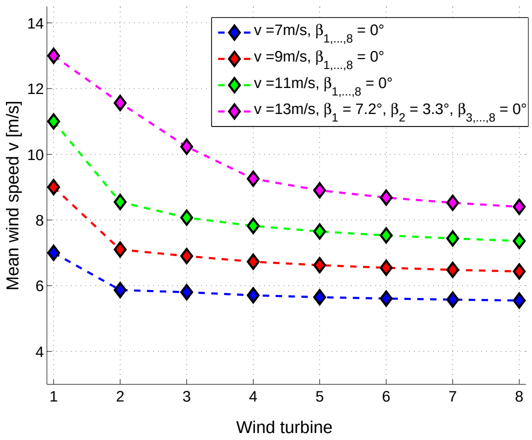

Fig. 3. Mean wind speeds at wind turbines

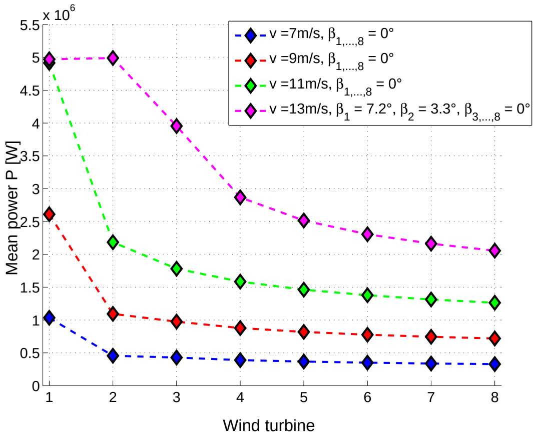

Fig. 4. Mean turbine power at various wind speeds

When there is coupling between the wind turbines it is quite possible that the control tactics that is optimal for a single turbine no longer renders the maximal power output. In this section we test this hypothesis.

A. Problem analysis

The wind farm model described in Section II is composed of eight turbines and contains a large number of free parameters. Furthermore, the system is described with implicit equations and contains look-up tables for turbine coefficients. Therefore, it is hard to analyse this problem. Instead, we first look at some smaller examples to deliver the illustrative examples

First we consider the system containing only two wind turbines in a row. By changing the tip-speed ratio of the last turbine and thus effecting its wake no increase in power can be achieved. But if we change the tip-speed ratio of the first turbine and weaken its wake we might obtain more power at the second turbine. Therefore we set the second turbine to operate at optimal tip-speed and change the tip-speed ratio of the first turbine. The Figure 5 shows the obtained power surplus (difference between the obtained power and the power obtained when both turbines operate with $\lambda = 7.5$) as a function of the tip speed ratio of the first turbine. It can be seen that in this case the power surplus is negative, so it is not possible to gain any power surplus by changing the tip-speed ratio of the first turbine. The second turbine can not compensate the power drop on the first turbine despite its increased wind speed.

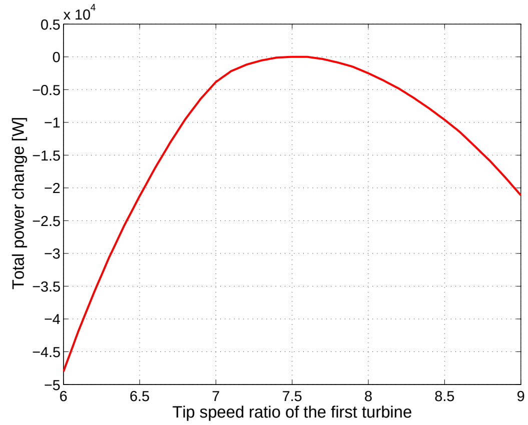

Fig. 5. System of two wind turbines - first turbine influence

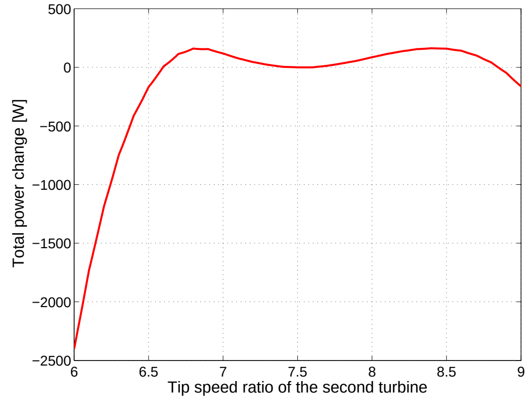

Fig. 6. System of three wind turbines - second turbine influence

However, if we add the third turbine at the beginning of the row and only keep its tip-speed ratio at the nominal value we are getting an entirely different picture. The result of such simulation is shown in Figure 6. It can be seen that surplus of power can be achieved. This demonstrates that there is the possibility of increasing the power production of the wind farm by coordinated control of the entire wind farm, which considers the wake coupling of wind turbines. However, we also notice the nonlinear character of the problem at hand. Even in this simple case the function of power surplus is nonconvex and therefore it is very hard to locate the global optimum.

B. Problem setup

For the general wind farm setup one can not find the optimum by enumeration, as it is done in the simple demonstration cases in the previous section.

The goal of the optimization problem is to find the rotor speed of every wind turbine for which the wind farm achieves maximum power. The problem, for n turbines, is set as follows:

$$\begin{aligned} &\max_{\omega_1,\ldots,\omega_n} && \sum_{i=1}^{n} P_i(\omega_i) \\ &\text{subject to} && \begin{cases} w_{min} \leq w_i \leq w_{max} \\ 0 \leq P_i \leq P_{max} \\ \dfrac{\partial C_q}{\partial \lambda} < 0 \\ (1), (6), (7). \end{cases} \end{aligned} \tag{8}$$

There are several constraints which must be met. The rotor speed ω must be between minimum allowed and rated. The maximum power that turbine can provide must be positive and less or equal to rated P_{max} = 5 MW. To ensure that the operating point is stable, the torque coefficient (C_q) curve must have negative slope in that point [1].

This optimization problem is a nonconvex, both in the cost function and in the constraints. There is no readily available solver that can handle such optimzation problems and guarantee that the achieved optimal point is indeed the global minimum of the problem [5].

The problem is solved by using the sequential quadratic programming algorithm implemented in Matlab function `fmincon`, [7]. To increase the probability of finding the global minimum of the problem at hand we use the `GlobalSearch` function implemented in Matlab, [6]. This function uses several iterations of `fmincon` to find the best solution.

C. Results

The results obtained by using proposed functions and algorithms are shown in Figure 7. The states (rotor speed, tip speed ratio, wind speed, power) of optimally controlled wind farm are compared with the states of traditionally controlled one. The main difference in control approach is noticeable in comparison of tip speed ratios. The tip speed ratios of every turbine in traditionally controlled (non-optimized) wind farm have the same value of 7.5, for which every turbine works at its optimal operating point. In the optimized wind farm, only the last turbine in a row works at its optimal operating point because it does not effect any other turbine. As the result, wind speed on every turbine, after the second one, is increased. The first turbine remains the same power production. Even though it has maximum impact on speed deficit in wakes, it also has the largest share in the wind farm power production, so there is no benefit in reducing its power. The second and the third turbine decrease their power production and minimize wakes behind them. The power gains on other turbines results in an increase in total wind farm power production. The most power surplus is generated on the last two turbines, they seize the power the turbines in front saved. The detailed results can be seen in Table I.

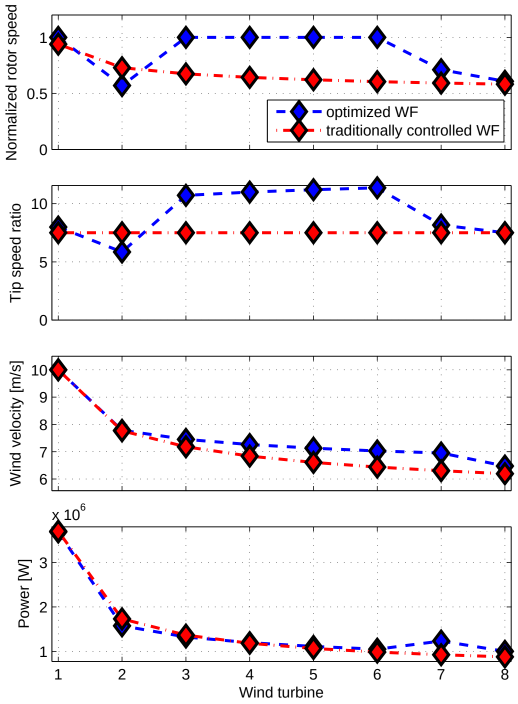

Fig. 7. Simulation results for mean wind speed 10 m/s

TABLE I
RESULTS FOR MEAN WIND SPEED 10 M/S

	P_{WF} [kW]	P_{WT} [kW]
Traditional control	11844	3699 1732 1368 1184 1068 987 927 879
Optimal control	12182 +2.85%	3683 -0.44% 1575 -9.03% 1324 -3.23% 1198 +1.23% 1112 +4.13% 1052 +6.52% 1233 +33.09% 1004 +14.17%

IV. LOAD OPTIMIZATION

At the wind speeds larger than rated all turbines work at rated rotor speed and they are controlled by changing the blade pitch angle. There is enough wind energy available and every turbine can work at its rated power, which is due to mechanical and electrical constraints the largest power it can permanently produce, so there is no opportunity for maximizing the power extraction. However, if the wind farm must follow the power reference that is lower than the rated power, the load optimization may be considered. This operating mode is used when the wind farm is obliged to participate in grid regulation, or it can not deliver its maximum power due to faults in transmission lines.

The loads considered here are wind turbine structural loads, particulary the tower bending moment and blade bending moment. The tower bending moment is the sum of the moments caused by the thrust on the rotor, the aerodynamic force on the tower, and the eccentricity of the nacelle. The blade bending moment originates from the axial force on the rotor blade and the tangential force due to gravity.

A. Problem analysis

The goal of a load optimization can be minimizing total load of the entire wind farm. However, that goal is very hard to achieve without sacrificing the power.

We consider different type of optimization where the load is evenly distributed among the turbines. In the case when each turbine produces the same power very large difference between the turbine loads emerges due to wake effects. This leads to uneven wear and tear of the turbines and therefore possible malfunctions, maintenance requirements or shorter operating life. We aim to reallocate the power in the wind farm such that every turbine suffers the same amount of the load.

B. Problem setup

The problem that equalizes the turbine loads while the wind farm follows the power reference is set as follows:

$$\begin{aligned}
\min_{\beta_1,\ldots,\beta_n} \quad & \sum_{i=1}^{n-1} \lambda_{\mathrm{b}}(M_i^{\mathrm{b}}(\beta_i) - M_{i+1}^{\mathrm{b}}(\beta_i))^2 + \\
& + \lambda_{\mathrm{t}}(M_i^{\mathrm{t}}(\beta_i) - M_{i+1}^{\mathrm{t}}(\beta_i))^2 + (P_{ref} - P_{total})^2 \\
\text{subject to} \quad & \begin{cases} \beta_{min} \le \beta_i \le \beta_{max} \\ P_i \le P_{max} \\ \dfrac{\partial C_q}{\partial \lambda} < 0 \\ (2), (5), (7) \end{cases}
\end{aligned} \tag{9}$$

The average tower bending moment at the i-th turbine M_i^t and the corresponding blade bending moment M_i^b are obtained from the expressions in [2]. Those expressions determine the average values of the moments as function of axial induction factor under the preassumption of standard distribution. The scaling factors λ_{t} and λ_{b} are used to balance between different requirements.

This problem is also solved with Matlab function `fmincon` [6]. The active-set algorithm, which proved to be better in this case than the `sqp`, was used.

C. Results

The simulation results for the wind speed of 15 m/s are shown on the figure 8. The blade pitch angle, tower and blade bending and power of the turbines with equalized loads are compared with the traditionally controlled wind farm where each wind turbine operates at power reference $\frac{P_{ref}}{N}$, where N is number of turbines. The significant power and load reallocation was achieved with minor changes in the blade pitch angles. In the power equalized wind farm, the lowest loads has first turbine in a row and the last has the highest. To equalize the loads it was necessary to decrease blade pitch angle of the first turbines, which increased their efficiency, and to increase blade pitch angle of the last few turbines.

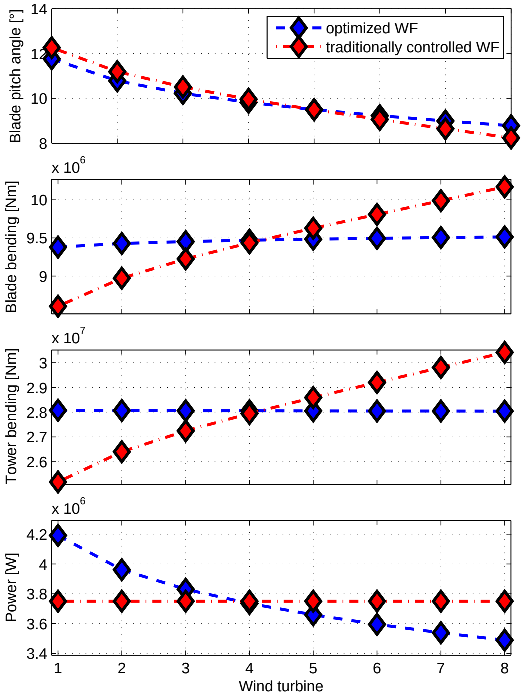

Fig. 8. Simulation results for mean wind speed 15 m/s

A detailed overview of the power and load is shown in the Table II. The table only lists the changes in the tower bending moment, while the changes of the blade bending moment are very similar. The link between percentage change of power and load is noticeable. In both cases, the wind farm produces power equal to the given power reference of 30 MW.

V. CONCLUSION

In this paper the optimization of wind farm optimization based on wind farm modeling is investigated. The wind farm model is used to describe the coupling between turbines that occurs due to wakes.

Two different optimization objectives are used based on the operating regime in which the wind farm operates. At lower wind speeds, the goal is to maximize the produced power. By taking into account the effect of turbine coupling the gain of 2.85% in the produced power was obtained.

TABLE II
RESULTS FOR MEAN WIND SPEED 15 M/S

	M_{WF} [MNm]	M_{WT} [MNm]	P_{WT} [kW]
Equalized power	224.8	25.2 26.4 27.2 27.9 28.6 29.2 29.8 30.4	3750 3750 3750 3750 3750 3750 3750 3750
Equalized load	224.5 -0.15%	28.1 +11.5% 28.1 +6.3% 28.1 +3.0% 28.1 +0.4% 28.1 -1.9% 28.0 -3.9% 28.0 -5.9% 28.0 -7.8%	4191 +11.8% 3962 +5.6% 3830 +2.1% 3735 -0.4% 3659 -2.4% 3595 -4.1% 3539 -5.6% 3489 -6.9%

At higher wind speeds where the nominal wind farm power is achieved, the goal was to optimize wind farm loads. In traditional wind farm control there are the turbines that are more exposed to loads and therefore also to increased wear and tear. By reallocating the power production the loads in the wind farm are equalized and the high loads on certain turbines are evaded.

ACKNOWLEDGEMENT

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