Bearing-Only Tracking with a Mixture of von Mises Distributions*

Ivan Marković[1] and Ivan Petrović[2]

Abstract— **This paper presents a novel method for Bayesian bearing-only tracking. Unlike the classical approaches, which involve using Gaussian distribution, the tracking procedure is completely covered with the von Mises distribution, including state representation, transitional probability, and measurement model, since it captures and models well the peculiarities of directional data. The state is represented with a mixture of von Mises distributions, thus offering advantages of being able to model multimodal distributions, handle nonlinear state transition and measurement models, and to completely cover the whole state space, all with a modest number of parameters. The tracking procedure is solved by convolution with a von Mises distribution (prediction step) and multiplication with a mixture representing the measurement model (update step). Since in the update step the number of mixture components grows exponentially, a method is presented for component reduction of a von Mises mixture. Furthermore, a closed-form solution is derived for quadratic Rényi entropy of the von Mises mixture. The algorithm is tested and compared to a particle filter representation in a speaker tracking scenario on a synthetic data set and real-world recordings. The results supported the proposed approach and showed similar performance to the particle filter.**

I. Introduction

Directional data, like bearing (azimuth) and heading, is encountered often in many applications, including mobile robotics. For an example, the heading direction in odometry, compass measurements, bearing of various features in monocular camera images (both perspective and catadioptric), and the speaker bearing estimated with a microphone array, to name but a few. Working with directional data, especially under uncertainty, imposes a problem on how to represent them in probabilistic frameworks. Commonly this problem is solved by using a Gaussian distribution, which unfortunately does not capture well the non-euclidean properties of directional data. Furthermore, since small errors in the heading can result with great errors in the final location, the need to faithfully model the directional data should not be dismissed lightly. Therefore, circular distributions, of which von Mises is an example, are often adopted and utilized to model angular random variables.

The robotics community has recognized the benefits of the von Mises distribution to model directional data. In [1] the von Mises distribution was used in odometry to deal with the heading changes for topological model learning. In [2] the authors proposed a solution for solving large-scale partially observable Markov decision processes and tested the algorithm first on a synthetic problem of a circular corridor, where the transition and observation probabilities were modeled with the von Mises distribution. In [3] the emission distribution of a hidden Markov model was learned by estimating parameters of the von Mises distribution in order to model compass measurements in a localization problem. In our previous work [4] we also utilized the von Mises distribution for speaker localization and tracking, but only to model the measurement likelihood of the microphone array, whereas the particle filter (PF) was used for the state representation.

In the present paper, we propose to model the complete bearing-only tracking process with the von Mises distribution; from the state representation and transition probability to the measurement likelihood. Compared to the PF, the benefits of the proposed approach lie in representing the function and not just the density, and in the fact that less components are needed to model the state. For the classical Bayesian tracking procedure with a mixture of von Mises densities we solved the following problems: (i) the convolution and (ii) the product of two von Mises distributions, (iii) the algorithm for component reduction of a mixture of von Mises distributions, and (iv) the analytical expression for the entropy of a mixture of von Mises distributions in order to have a measurement of the state uncertainty. The solution for the first two problems are presented from the literature, the third problem is solved by adapting a component reduction technique for Gaussian distributions, while the fourth problem is solved by deriving the entropy from the beginning.

The rest of the paper is organized as follows. In Section II we present the theoretical background required for Bayesian tracking with a mixture of von Mises distributions. Section III reports the results of experiments with synthetic and real-world data, while Section IV concludes the paper.

II. Theoretical background

The problem at hand is to analyze and make inference about a dynamic system. For that, two models are required: one predicting the evolution of the state over time (system model), and one relating the noisy measurements to the state (measurement model). We assume that both models are available in probabilistic form. Thus, the approach to dynamic state estimation consists of constructing the *a posteriori* probability density function (pdf) of the state based on all available information.

A Bayesian tracking procedure consists of two steps: prediction and update [5], [6]. The prediction step involves

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[1,2]I. Marković and I. Petrović are with University of Zagreb, Faculty of Electrical Engineering and Computing, Department of Control and Computer Engineering, HR-10000 Zagreb, Croatia (ivan.markovic, ivan.petrovic) at fer.hr

calculating the prior pdf via the total probability theorem:

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}) = \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})\, d\boldsymbol{x}_{k-1}, \tag{1}$$

where $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ is the probabilistic model of the state evolution, $p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})$ is the posterior at time $k-1$, and $\boldsymbol{z}_{1:k-1}$ are all measurements up to and including time $k-1$. In the update step, the posterior at time k is calculated via the Bayes theorem:

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k}) = \frac{p(\boldsymbol{z}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})}{p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1})}, \tag{2}$$

where $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ is the sensor model and $p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1})$ is the normalizer, which can be calculated using the total probability theorem via:

$$p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1}) = \int p(\boldsymbol{z}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})\, d\boldsymbol{x}_k. \tag{3}$$

In this paper we infer about circular random variables and that brought us to the use of the von Mises distribution for modeling the system state, state transition probability, and sensor model. Given that, we now explicitly calculate the relations (1) and (2) for von Mises distributions.

A. The von Mises distribution

The pdf of the von Mises distribution is given by [7]:

$$p(x; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} \exp\left[\kappa \cos(x-\mu)\right], \tag{4}$$

where $0 \leq x \leq 2\pi$, μ is the mean direction, $\kappa \geq 0$ is the concentration parameter, and

$$I_n(\kappa) = \frac{1}{2\pi}\int_0^{2\pi} \exp(\kappa\cos\xi)\cos(n\xi)\, d\xi \tag{5}$$

is the modified Bessel function of the first kind and of order n. The distribution is unimodal and symmetric around the μ. The mean direction μ is analogous to the mean of the Gaussian distribution, while the concentration parameter κ is analogous to the inverse of the variance of the Gaussian distribution.

B. Convolution of the von Mises distributions

The prediction step given by (1) is actually a convolution of two pdfs. Given two von Mises pdfs, $p(x; \mu_i, \kappa_i)$ and $p(x; \mu_j, \kappa_j)$, the resulting convolution of a predicted state will be of the following form [8]:

$$h(x) = \frac{1}{2\pi I_0(\kappa_i) I_0(\kappa_j)} \cdot I_0\left(\left\{\kappa_i^2 + \kappa_j^2 + 2\kappa_i\kappa_j + \cos(x - [\mu_i + \mu_j])\right\}^{1/2}\right), \tag{6}$$

which in fact is not a von Mises distribution, but can be well approximated by the following pdf [8]:

$$h(x) \approx p(x; \mu_i + \mu_j, A^{-1}(A(\kappa_i)A(\kappa_j)) \tag{7}$$

where

$$A(\kappa) = \frac{I_1(\kappa)}{I_0(\kappa)},\ \kappa \geq 0 \tag{8}$$

is the ratio of the modified Bessel functions of order one and order zero, and $A^{-1}(\,.\,)$ is its inverse.

C. Product of the von Mises distributions

The numerator in the update step given by (2) involves a calculation of a product of two von Mises distributions. Given the following von Mises pdfs, $p(x; \mu_i, \kappa_i)$ and $p(x; \mu_j, \kappa_j)$, the resulting product is of the following form [9]:

$$g(x) = \frac{1}{4\pi^2 I_0(\kappa_i) I_0(\kappa_j)} \exp\left[\kappa_{ij}\cos(x - \mu_{ij})\right], \tag{9}$$

where

$$\mu_{ij} = \mu_i + \operatorname{atan2}\left(-\sin\Delta\mu, \kappa_i/\kappa_j + \cos\Delta\mu\right), \tag{10}$$

$$\kappa_{ij} = \sqrt{\kappa_i^2 + \kappa_j^2 + 2\kappa_i\kappa_j\cos\Delta\mu}, \tag{11}$$

and $\Delta\mu = \mu_i - \mu_j$. The product in (9) is an unnormalized von Mises distribution. Note that in order to complete the update step, we still need to calculate (3). This step can be circumvented since (9) can be well approximated by the following von Mises pdf [9]:

$$g(x) \approx p(x; \mu_{ij}, \kappa_{ij}), \tag{12}$$

where the mean direction, μ_{ij}, and the concentration parameter, κ_{ij}, are given by (10) and (11), respectively.

It is interesting to note at this point that the product of von Mises distributions calculated by (9) has very different properties than the product of Gaussian distributions. For an example, the concentration parameter of the product is a function of the factor pair mean directions and concentrations, while in the case of Gaussian distributions, the variance of the product is only function of the factor pair variances. Given that, if the distance between factor pair mean directions is large enough, it is possible that the concentration parameter of the product will be smaller (representing higher uncertainty) than any concentration parameter of the factor pair. Indeed, a product of von Mises distributions calculated via (12) with equal concentration parameters and antipode mean directions will yield a uniform distribution.

D. Tracking with a mixture of von Mises distributions

In this paper, the goal is to estimate the bearing, x_k, of the tracked object at time k given the posterior $p(x_k|\boldsymbol{z}_{1:k})$. Note that x_k is now a scalar value, but that the measurements, $\boldsymbol{z}_k$, can still be a vector. There exists many approaches to state representation and state estimation [5], [10], some of them being Gaussian, histogram, particle, Gaussian mixture and kernel mixture representations. State estimation from all of these approaches follows the classical Bayesian approach of prediction-update steps, which for the Gaussian representation are explicit in the form of the Kalman Filter and its variants. For the rest, the reader is directed to [5], [10] for reference.

Previously stated representations, albeit excluding the (single) Gaussian representation, have the ability to model multi-modal pdfs and deal quite well with highly non-linear functions. However, most of them are appropriate for estimating states of euclidean nature. Take for an example, the problem of maximum likelihood (ML) estimation of the mean value of independent identically distributed Gaussian random variables. For such a problem, the solution would be to calculate the arithmetic mean [11]. However, if such approach was applied to angular values θ, ranging from 0 to 2π, then the solution would yield an incorrect result, since the mean value of just 0 and 2π would be π instead of 0 or 2π. Moreover, values such as $\theta \pm 2k\pi, k \in \mathbb{N}$ should all have equal probabilities.

Although there are several distributions appropriate for circular models [8], the von Mises distribution is the most commonly used and studied, since it provides a closed-form analytical framework for many applications. For an example, the ML estimator of the mean of independent identically distributed von Mises random variables would yield [8]:

$$\hat{\mu} = \operatorname{atan2}\left[\sum_i \sin(\mu_i), \sum_i \cos(\mu_i)\right], \tag{13}$$

which is the correct expression for calculating the mean of angular values. Given that, we represent the posterior of a circular random variable as a convex combination of N von Mises kernels:

$$p(x_k|\boldsymbol{z}_{1:k}) = \sum_{i=1}^{N} \gamma_i \frac{1}{2\pi I_0(\kappa_i)} \exp\left[\kappa_i \cos(x_k - \mu_i)\right], \tag{14}$$

where $\sum_i \gamma_i = 1$. As stated earlier, the state transition probability is also a von Mises pdf:

$$p(x_k|x_{k-1}) = \frac{1}{2\pi I_0(\kappa)} \exp\left[\kappa \cos(x_k - x_{k-1})\right], \tag{15}$$

which on closer inspection will only spread the posterior (increase the uncertainty) in the prediction step. Following a similar train of thought as for (14), we also write the sensor model as a convex combination of M von Mises pdfs:

$$p(\boldsymbol{z}_k|x_k) = \sum_{i=1}^{M} \gamma_i \frac{1}{2\pi I_0(\kappa_i)} \exp\left[\kappa_i \cos(x_k - z_{k,i})\right], \tag{16}$$

where $\sum \gamma_i = 1$, not necessarily equal to the ones in (14). Note that by doing so, we also allow the sensor model to be a multimodal pdf.

Finally, from a multimodal distribution we infer the state x_k as a maximum a posteriori (MAP) estimate from the posterior $p(x_k|\boldsymbol{z}_{1:k})$:

$$\hat{x}_k = \arg\max_{x_k} p(x_k|\boldsymbol{z}_{1:k}). \tag{17}$$

Basically, a Bayesian tracking algorithm with previously defined state representation, motion model and sensor model, would consist of: (i) initially setting up an a priori distribution via (14) (N von Mises pdfs uniformly spread with small κ), (ii) convolving (14) with (15), (iii) multiplying the result of the convolution with (16), (iv) estimating the state, and then repeating steps (ii), (iii), and (iv) over time. The only problem with the previous procedure is the step (iii), where the state representation consisting of N von Mises pdfs is multiplied with M von Mises pdfs of the sensor model. This yields $M \cdot N$ von Mises pdfs and would hence grow exponentially in time. In order to solve this problem, we need to reduce the number of the components in the mixture.

E. Reducing the number of mixture components

In this paper, we propose a variant of the West's algorithm [12] for reduction of the number of von Mises components, which in its original form has computational complexity of $\mathcal{O}(N \log N)$ [10]. West's algorithm, in essence, reduces the number of components by searching for the nearest neighbour, and then replaces the pair with a single component whose parameters are an average of the pair's values. Originally, this algorithm was developed to reduce the number of components with equal variances, with similarity criteria being the nearest neighbour in the mean value. In order to adapt the algorithm for reducing the mixture of von Mises components, we introduce the following modifications.

The most important modification is the use of Bhattacharyya coefficient as a measure of pdf similarity:

$$c_B(p, q) = \int_0^{2\pi} \sqrt{p(\xi) q(\xi)}\, \mathrm{d}\xi, \tag{18}$$

where $0 \leq c_B \leq 1$, and can be though of as a measure of overlap of two pdfs (0 if no overlap). Another practical property of c_B is that it can be derived for two von Mises pdfs in closed form [13]:

$$c_B\left(p(x;\mu_i,\kappa_i), p(x;\mu_j,\kappa_j)\right) = \frac{I_0\left(\kappa_{ij}/2\right)}{\left\{I_0(\kappa_i) I_0(\kappa_j)\right\}^{1/2}}. \tag{19}$$

In [13] c_B was derived for expectation-maximization (EM) algorithm in order to estimate the parameters of a mixture of von Mises distributions. Similarly, the EM approach could be applied in this paper for component number reduction, but the computational complexity would be higher than that of the West's algorithm [10]. The rest of the modifications are minor, and the pseudocode is given in Algorithm 1.

F. Entropy of the von Mises mixture

In tracking applications it is often very practical, if not necessary, to have a measure of uncertainty of the tracked state. While the uncertainty of unimodal distributions is characterized by their respective parameters, for multimodal distributions the same it is not that straightforward. Therefore, for the latter case, entropy is usually utilized for uncertainty calculation and as a practical feature in track management [10], [14].

Entropy of a mixture of von Mises-Fisher distributions, a probability distribution on a sphere, can be found in [15]. However, reducing the dimension of the result in [15] in order to derive an expression for entropy of a mixture of von Mises distributions is not a straightforward task, and therefore we derive a closed-form solution in this paper. Note that in this derivation we allow the kernels to have different concentrations parameters.

Algorithm 1 Reduction of the von Mises components

Require: Components parameters $\{\mu_i, \kappa_i, w_i\}_{i=1}^{NM}$
Ensure: Reduced component parameters $\left\{\mu_j^*, \kappa_j^*, w_j^*\right\}_{j=1}^{N}$

```
Sort ascending by weights: sort({μ_i, κ_i, w_i}_{i=1}^{NM})
r ← MN
while r > N do
    for i = 2 : r do
        c_B ← I_0(κ_{1i}/2) / {I_0(κ_1) I_0(κ_i)}^{1/2}
    end for
    k ← find(c_B == max(c_B))
    Remove components 1 and k:
    remove({μ_i, κ_i, w_i}_{i=1,k}, {μ_i, κ_i, w_i}_{i=1}^{r})
    μ* ← μ_1 + atan2[−sin(μ_1 − μ_k), κ_1/κ_k + cos(μ_1 − μ_k)]
    κ* ← max(κ_1, κ_k)
    w* ← w_1 + w_k
    Insert new component by weight:
    insert({μ*, κ*, w*}, {μ_i, κ_i, w_i}_{i=1}^{r−2})
    r ← r − 1
end while
{μ_j*, κ_j*, w_j*}_{j=1}^{N} ← {μ_i, κ_i, w_i}_{i=1}^{N}
```

A measure of entropy can take many analytical forms. Shannon entropy of a mixture of distributions cannot be expressed in closed-form, while Rényi entropies usually offer a more suitable framework for analytical calculations [16]. Therefore, to calculate entropy of the von Mises mixture, we used the Rényi entropy, which of order α is defined as follows [17]:

$$H_\alpha(x) = \frac{1}{1-\alpha} \log \int p^\alpha(x)\,\mathrm{d}x, \tag{20}$$

where $1 \leq \alpha < \infty$. In the limit $\alpha \to 1$ Rényi entropy becomes Shannon entropy.

The quadratic Rényi entropy of a von Mises mixture is derived as follows:

$$\begin{aligned} H_2(x_k) &= -\log \int_0^{2\pi} p^2(x_k|\boldsymbol{z}_{1:k})\,\mathrm{d}x_k \\ &= -\log \int_0^{2\pi} \left(\sum_{i=1}^{N} \frac{\gamma_i \exp\left[\kappa_i \cos(x-\mu_i)\right]}{2\pi I_0(\kappa)} \right)^2 \mathrm{d}x_k \\ &= -\log \int_0^{2\pi} \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\gamma_i \exp\left[\kappa_i \cos(x_k-\mu_i)\right]}{2\pi I_0(\kappa_i)} \\ &\quad \cdot \frac{\gamma_j \exp\left[\kappa_j \cos(x_k-\mu_j)\right]}{2\pi I_0(\kappa_j)}\,\mathrm{d}x_k \\ &= -\log \int_0^{2\pi} \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\gamma_{ij} \exp\left[\kappa_{ij} \cos(x_k-\mu_{ij})\right]}{4\pi^2 I_0(\kappa_i) I_0(\kappa_j)}\,\mathrm{d}x_k, \end{aligned} \tag{21}$$

where $\gamma_{ij} = \gamma_i\gamma_j$, and μ_{ij} and κ_{ij} are given by (10) and (11), respectively. By rearranging the sums and the integral, and by using definition (5) for $n = 0$, we arrive to the final expression for the quadratic Rényi entropy:

$$\begin{aligned} H_2(x_k) &= -\log \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\gamma_{ij}}{2\pi I_0(\kappa_i) I_0(\kappa_j)} \\ &\quad \cdot \frac{1}{2\pi} \int_0^{2\pi} \exp\left[\kappa_{ij}\cos(x_k-\mu_{ij})\right]\mathrm{d}x_k \\ &= -\log \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} \frac{I_0(\kappa_{ij})}{2\pi I_0(\kappa_i) I_0(\kappa_j)}. \end{aligned} \tag{22}$$

Note that in the last step we have lost explicit dependence on x_k. But on closer inspection, we can see that the state is implicitly included in κ_{ij} through the difference $\Delta\mu = \mu_i - \mu_j$.

We can also utilise the symmetry $\kappa_{ij} = \kappa_{ji}$ in order to reduce the number of terms in the double sum in (22):

$$\begin{aligned} H_2(x_k) = -\log \frac{1}{2\pi} &\left[\sum_{i=1}^{N} \frac{I_0(2\kappa_i)}{I_0^2(\kappa_i)} \right. \\ &\left. + 2\sum_{i=2}^{N}\sum_{j=1}^{i-1} \frac{I_0(\kappa_{ij})}{I_0(\kappa_i) I_0(\kappa_j)} \right]. \end{aligned} \tag{23}$$

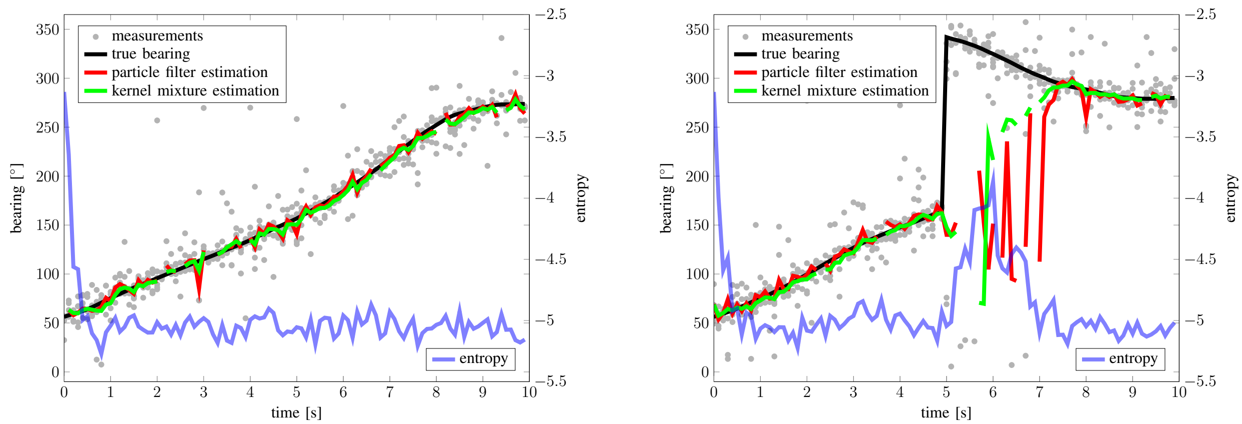
III. Experiments

In this section we investigated the application of the proposed algorithm in a bearing-only tracking scenario. Furthermore, the proposed algorithm was compared to an algorithm based on (PF) from a former paper [4]. Without getting into details on the signal processing and how the bearing measurements are derived, only the concept of the used sensor model is presented.

Basically, if we have two passive sensors like microphones, we can estimate the phase shift between the two recorded signals. From this phase shift and known distance between the microphones, one can estimate the bearing of the sound source. Commonly, this phase shift is determined via time difference of arrival (TDOA) procedure, where the signals are cross-correlated and the maximum peak is searched for. However, front-back ambiguity is inherent to this procedure, since sound sources emanating in front of and behind the sensor pair will have the same phase shift. This problem is solved by using more than two non co-linear sensors, and the idea in [4] was to model the measurements of the sensor array as a mixuture of von Mises distributions, thus yielding a multimodal sensor model like (16). The important thing to realize from this paragraph is that while tracking a single target, we will work with a multimodal distribution representing the sensor measurements. Naturally, this procedure can also be utilized in a classical scenario when a sensor reports a unimodal distribution. For details on the speaker localization and tracking algorithm, the reader is directed to [4].

A. Synthetic data

We have simulated two trajectories of a maneuvering object in 2D, where the dynamics of the system were described by a jump-state Markov model [18]. The second



(a) The first trajectory

(b) The second trajectory

Fig. 1. Bearing estimation for the two simulated trajectories. For the first trajectory we can see that both estimators have similar performance—root mean square error (RMSE) was 2.7° and 2.8° for the von Mises mixture estimator and PF, respectively. The second trajectory depicts the turn-taking scenario. We can see that again both show similar performance, and were a bit reluctant at the beginning to switch to a new bearing value. Concerning the entropies, at the beginning the entropy is largest since the distribution is close to uniform. As the filter is updated with measurements the entropy drops. We can also see the result of the turn-take at 5–7 s in the second trajectory where the entropy rose due to discrepancy between the believed state and measurements.

trajectory had a rapid change in the bearing value to simulate a turn-taking scenario in order to test the capability of the algorithms to keep up with the track in such situations. For an example, this might occur when one speaker stops talking and the other continues, or the currently talking speaker stops, moves around the robot and then continues talking again. Note that the application of the described speaker localization algorithm is to detect and track the currently active speaker, and not to to detect and track multiple concurrently talking speakers and keep separate tracks for each one.

In order to make the simulation as realistic as possible (i) measurements were corrupted with von Mises noise of $\kappa = 70$ to model measurement noise, (ii) outliers where added with probability $P_O = 0.3$, i.e. close to 30% of measurements at random locations were corrupted with von Mises noise of $\kappa = 5$, and (iii) detection probability was $P_D = 0.9$, i.e. close to 10% of measurements at random locations were discarded.

For the von Mises mixture estimator, we used 12 components with mean directions uniformly spread over 0 to 2π, the process model was a single von Mises pdf, while the likelihood consisted of 12 components. The state was always represented with 12 kernels but concentration parameters changed at each iteration. Note that in the case of a mixture of von Mises pdfs, due to (11), it is not possible to work with components of the same concentration parameter since the updated κ is a function of the mean directions.

The PF was implemented as described in [4], where the likelihood also consisted of 12 von Mises pdfs, the state was represented with 360 particles, and the process model was a Langevin motion model [19]. Instead of resampling we used a variant of regularization [20], where we placed a von Mises kernel on each particle instead of a Gaussian distribution and drew new particles from such a multimodal distribution.

The results of the bearing estimation of both trajectories with the mixture of von Mises pdfs and with the PF along with corresponding entropies are shown in Fig. 1. In Fig. 1(b) we can see that the tracking algorithms did not switch right away to the new bearing value. Of course, both could be tuned to respond faster to rapid changes by decreasing κ of the transition pdf or by increasing κ of the measurement likelihood, but this would be at the cost of higher sensitivity to outliers. The former parameter tuning depends on the characteristics of the sensor measurements—if we expect large percentage of outliers, then we should make the estimator more inert, and vice-versa.

The number of parameters required for the state representation was smaller in the case of the mixture filter. We used 12 kernels, i.e. 36 parameters including the means, concentration parameters, and weights, while for the particle filter we used 360 particles, i.e. 360 parameters after regularization (due to equal particle weights).

The execution time was measured for Matlab implementation on an Intel Core2Quad processor with 2.33 GHz frequency (only one core was used). The mean time of an iteration was 81.2 ms and 72.5 ms for the mixture filter and the PF, respectively. Furthermore, the mean time of the component reduction algorithm was 19.7 ms, while the mean time of the regularization step of the particle filter was 68.2 ms. We can see that most of the execution time of the PF went to regularization. If sequential importance resampling (SIR) was utilised, it would significantly reduce the execution time (SIR took only 2.3 ms), but comparison of such a resampling procedure and regularization is out of the scope of this paper.

B. Real-world data

Microphone array, of our design, consisting of four omnidirectional microphones was placed on a Pioneer 3DX robot [4]. The audio interface was composed of low-cost microphones, preamplifiers and an external USB soundcard. The recordings were made with sampling frequency $F_s = 48$ kHz

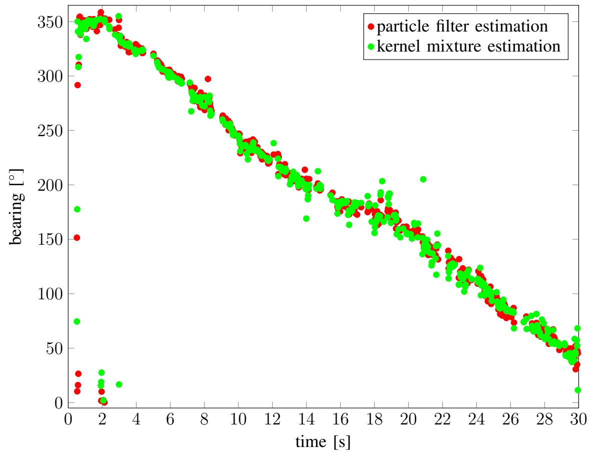


Fig. 2. Real-world data tracking of a speaker making a full circle around the microphone array. We can notice some outliers due to uniformity of the prior distribution at the initialization and corrupted measurements caused by difficult acoustic conditions (reverberation).

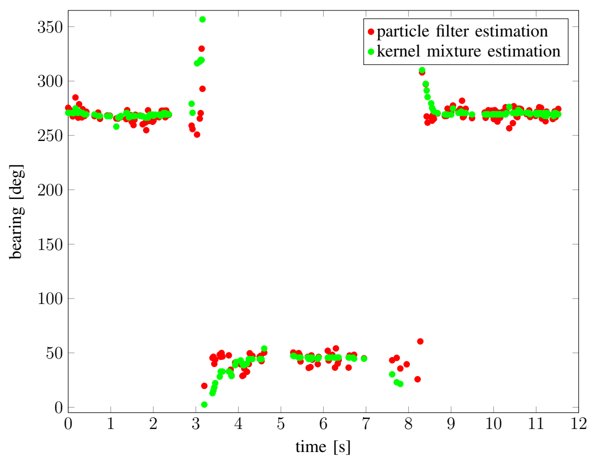


Fig. 3. Real-world data tracking of a turn-take scenario. The first speaker was located at $50°$, while the second was at $270°$. Qualitatively, the mixture filter here showed better results since it was able to follow the rapid change in the tracked value and maintain less noisy estimations.

and frame length $L = 1024$ samples in a classroom which has dimensions of $7\,\mathrm{m}\times 7\,\mathrm{m}\times 3.2\,\mathrm{m}$, parquet wooden flooring, and one side covered with windows. During the experiments, typical noise conditions were present, like computer noise and air ventilation.

Figure 2 shows the results of real-world tracking of a single speaker making a full circle around the microphone array, while Fig. 3 shows a turn-take scenario. Speakers were, at an approximate distance of $2\,\mathrm{m}$, reading sentences from the IEEE sentence database [21].

IV. Conclusion

We have presented all the theoretical steps of Bayesian tracking with a mixture of von Mises distributions. Although the algorithm was presented on the problem of speaker tracking with a microphone array, the potential field of interest is by no means limited to this application. The proposed approach can be utilized in any tracking scenario which involves bearing-only measurements. Furthermore, the paper highlights the merits of using a von Mises distribution for directional data, which does not receive that much attention due to pervasive use of the Gaussian distribution. One of the potential expected practical significances lies in systems where the communication bandwidth is limited, e.g. in decentralized architectures when different robots need to communicate a posteriori distributions.

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