Direction-only tracking of moving objects on the unit sphere via probabilistic data association

Ivan Marković, Mario Bukal, Josip Ćesić, and Ivan Petrović
University of Zagreb,
Faculty of Electrical Engineering and Computing
Unska 3, 10000 Zagreb, Croatia

Abstract—**Directional data can emerge in many scientific disciplines due to the nature of the observed phenomena or the working principles of a sensor. Such direction-only sensors can be used in applications with the aim of tracking multiple moving objects. One of the reasons why multiple moving object tracking can be challenging is because of the need to deal with the problem of pairing sensors measurements with tracked objects in the presence of clutter (the data association problem). In this paper we propose to approach the problem of multiple object tracking in clutter with direction-only data by setting it on the unit sphere, thus tracking the objects with a Bayesian estimator based on the von Mises-Fisher distribution and probabilistic data association. To achieve this goal we derive the probabilistic data association (PDA) filter and the joint probabilistic data association (JPDA) filter for the Bayesian von Mises-Fisher estimator on the unit sphere. The final PDA and JPDA filter equations are derived with respect to the Kullback-Leibler divergence by preserving the first moment of the spherical distribution. The performance of the proposed approach is demonstrated in experiments with synthetic data where moving object trajectories were simulated and noisy observations obtained along with the clutter simulated as a Poisson process on the unit sphere.**

I. Introduction

Directional data can emerge in many scientific disciplines. Since the surface of the earth is approximately a sphere, such data arise readily in earth sciences, e.g. the location of the earthquake's epicenter, the paleomagnetic directions of the earth's magnetic pole etc. Furthermore, many astronomical observations are points on the celestial sphere and as such yield directional data. In robotics, measurements from various sensors, due to their nature of operation, yield direction-only information of the objects of interest, e.g. microphone arrays measure the sound source direction, perspective and omnidirectional cameras measure the direction of various features of interest in space, the heading of the mobile robot calculated via odometry is a directional variable in two dimensions etc. One of the possible applications of such sensors is to utilize them in order to track moving objects in the context of probabilistic representation of directional data.

Considering the tracking of moving objects, the goal of such a system is to estimate the trajectories in scenarios with noisy measurements, clutter or false alarms (measurements that falsely appear to originate from moving objects) and multiple moving objects. The duties of such a system are truly manifold, and in the past several seminal methods have been developed in order to tackle this problem wherein data association plays one of the crucial roles. To solve this problem the methods that can be used are the global nearest neighbor (GNN) which attempts to find the single most likely data association hypothesis at each scan [1], the probabilistic data association (PDA) filter for single object tracking and joint probabilistic data association (JPDA) filter for multiple object tracking where multiple hypotheses are formed after each scan and then these hypotheses are combined before proceeding further with the next scan [2], the multiple hypothesis filter (MHT) [3] where multiple data association hypotheses are formed and propagated from scan to scan [1]. Also, another method for tracking of multiple objects is the probability hypothesis density (PHD) filter [4] which estimates the number of objects in the scene but does not solve the data association problem by itself, however a solution has been presented in [5] for the Gaussian mixture PHD filter [6].

Considering the tracking of moving objects with direction-only sensors, it was proposed in [7] to utilize the von Mises-Fisher distribution, a distribution on the unit sphere, to model the system state and the sensors measurements after which a Bayesian estimator was developed for single object tracking. This method was used in our previous work [8] in order to track a single moving object detected by an omnidirectional camera on a mobile robot assuming only a single moving object in the scene, thus not offering a consistent method for dealing with multiple moving objecst nor false alarms/clutter. A global nearest neighbor (GNN), which in contrast to JPDA solves the data association by hard assignment, was applied in tracking multiple moving objects on the sphere in [9] and the Rényi α-divergence was used as a distance measure. The comparison of the GNN and JPDA filter is out of the scope of this paper and the interested reader is directed to [1]. Direction-only estimation and tracking within a Bayesian framework using a probability distribution over quaternions (3D rotations), namely the Bingham distribution, was used in [10], [11] and in [12] where, furthermore, a second-order filter was derived which included also the rotational velocity. These approaches, advocating the unit hypersphere as the appropriate filtering space, showed better performance of the Bingham filter compared to the (extended) Kalman filter. However, the normalization constant of the Bingham distribution, hence its partial derivative, cannot be computed in closed form, but this can be surmounted by caching techniques and interpolation. On the contrary, the von Mises-Fisher distribution does not require such techniques since the normalization constant and its partial derivative can be calculated in closed form, which will be of practical interest in the ensuing JPDA equations, but it will involve at one point a numerical inversion of a ratio of Bessel functions since the derived equations are transcendental. Note that the Bingham and the von Mises-Fisher distributions reside on different manifolds, namely the unit quaternion hypersphere and the unit vector sphere, respectively. It is also worth bringing attention to similar approaches, where Bayesian filters were developed based on the von Mises distribution, a circular distribution in 2D [11], [13]–[15].

In this paper we propose a probabilistic data association solution to the problem of tracking a single and multiple moving objects in clutter with direction-only sensors. We pose the problem on the unit sphere and utilize a Bayesian tracking algorithm that is based on a spherical distribution, namely the von Mises-Fisher distribution. To solve the data association problem we derive the PDA and JPDA filter equations for the aforementioned Bayesian von Mises-Fisher filter. This constituted (i) deriving the a posteriori probabilities of association events on the unit sphere which essentially weigh each hypothetical estimation and form a mixture of von Mises-Fisher densities, (ii) determining the final (single) component density as the result of the update in the PDA and JPDA filter by preserving the first moment of the spherical distribution (which is optimal in the Kullback-Leibler sense), and (iii) modeling the false alarms as Poisson processes on the unit sphere. The proposed algorithms were tested on a synthetic data set comprising of single and multiple-object scenarios where direction-only measurements were corrupted with noise and clutter.

The paper is organized as follows. Section II presents the general mathematical background and formulae for tracking on the sphere with the von Mises-Fisher distribution. Section III describes the proposed PDA and JPDA filtering approaches based on the von Mises-Fisher distribution. Section IV presents the results of the synthetic data experiments, while Section V in the end concludes the paper.

II. General Background

When considering directions in d dimensions, i.e. unit vectors in d–dimensional Euclidean space $\mathbb{R}^d$, one can represent them as points on the $(d-1)$–dimensional sphere $\mathbb{S}^{d-1}$ of unit radius. Thus, in our notation, 1–sphere is the unit circle in $\mathbb{R}^2$ and the 2–sphere is the surface of the unit ball in $\mathbb{R}^3$. In the sequel we introduce all the necessary constituents and discuss the basic paradigm of a tracking system on the unit 2–sphere using von Mises–Fisher distributions.

A. von Mises–Fisher distribution

Parametric probability distribution $\mathfrak{f}(\boldsymbol{\mu}, \kappa)$ defined on the unit $(d-1)$–dimensional sphere $\mathbb{S}^{d-1}$, whose probability density function (pdf) is given by

$$f(\boldsymbol{x}; \boldsymbol{\mu}, \kappa) = C_d(\kappa) \exp(\kappa \boldsymbol{\mu} \cdot \boldsymbol{x}), \quad \boldsymbol{x} \in \mathbb{S}^{d-1}, \tag{1}$$

is called *von Mises–Fisher (vMF) distribution* with parameters $\kappa \geq 0$ and $\boldsymbol{\mu} \in \mathbb{S}^{d-1}$ denoting the concentration and the mean direction, respectively. Expression

$$C_d(\kappa) = \frac{\kappa^{d/2-1}}{(2\pi)^{d/2} I_{d/2-1}(\kappa)} \tag{2}$$

is the normalization constant, where I_p denotes the modified Bessel function of the first kind and order p [16]. Density (1) is rotationally invariant around the mean direction and, analogously to the multivariate Gaussian distribution, characterized by the maximum entropy principle in the sense that it maximizes the Boltzmann–Shannon entropy under prescribed directional mean [16].

Von Mises-Fisher distributions constitute an *exponential family* [17] parametrized by the natural parameter $\boldsymbol{\theta} = \kappa \boldsymbol{\mu} \in \mathbb{R}^d$ and the log-normalizing function given by

$$F_d(\boldsymbol{\theta}) = -\log C_d(\|\boldsymbol{\theta}\|). \tag{3}$$

The minimal sufficient statistics is the identity map, $\boldsymbol{t}(\boldsymbol{x}) = \boldsymbol{x}$ on $\mathbb{S}^{d-1}$, hence, the vMF distribution is completely determined by the *directional (angular) mean* [1]

$$\mathbb{E}[\boldsymbol{x}] = \int_{\mathbb{S}^{d-1}} \boldsymbol{x} f(\boldsymbol{x}; \boldsymbol{\mu}, \kappa) \mathrm{d}\boldsymbol{x} = \nabla F_d(\boldsymbol{\theta}) =: A_d(\kappa)\boldsymbol{\mu}, \tag{4}$$

where $A_d(\kappa)$ is the ratio of the following Bessel functions

$$A_d(\kappa) = \frac{I_{d/2}(\kappa)}{I_{d/2-1}(\kappa)}. \tag{5}$$

Due to the present application of moving objects tracking, we are particularly interested in case of $d = 3$ and vMF distributions on the unit 2–sphere $\mathbb{S}^2$, where the above expressions simplify to [16]

$$C_3(\kappa) = \frac{\kappa}{4\pi \sinh \kappa} \quad \text{and} \quad A_3(\kappa) = \frac{1}{\tanh \kappa} - \frac{1}{\kappa}. \tag{6}$$

An example of a von Mises-Fisher distribution on the unit 2–sphere $\mathbb{S}^2$ with different mean directions and concentration parameters is depicted in Fig. 1.

B. Motion model

In our model we assume that moving objects are relatively slow with respect to the sampling rate, i.e. changes in the objects's position between two consequent observations are relatively small. Mathematically, motion of such objects is then described by a wide-sense stationary stochastic processes, among which, the Wiener process (Brownian motion) is the standard choice [18]. These time-continuous processes are typically further approximated by a random walk of a fixed time step. We will briefly describe two model examples whose transition probability function can be well approximated by the vMF distributions, thus motivating the nonlinear filtering framework discussed in Section II-D.

First, consider the isotropic Wiener process in $\mathbb{R}^3$ and corresponding time-discretization (random walk) of fixed time step $\tau > 0$. The transition probability density function of the process is given by the Gaussian density

$$p(\boldsymbol{x}^k | \boldsymbol{x}^{k-1}) = \frac{1}{(2\pi\sigma_\tau^2)^{3/2}} \exp(-\|\boldsymbol{x}^k - \boldsymbol{x}^{k-1}\|^2 / 2\sigma_\tau^2), \tag{7}$$

[1] Note that the *directional mean* is defined by the integral (4), while the *mean direction* $\boldsymbol{\mu}$ is the parameter of the von Mises-Fisher distribution.

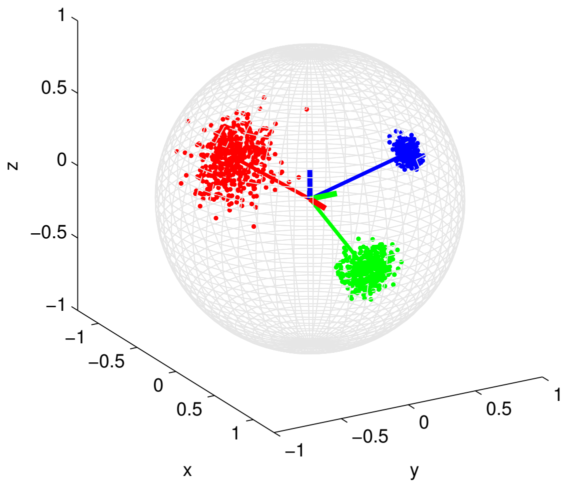


Fig. 1: Samples on the unit 2–sphere of the von Mises-Fisher distribution with different mean directions and concentration parameters of 50 (red), 150 (green), 500 (blue).

where $\sigma_\tau^2 := \sigma^2\tau$ and $\sigma > 0$ denotes the process strength. If we are confined to a mesurement device which only measures direction $\hat{\boldsymbol{x}}^k \in \mathbb{S}^2$ of position vectors $\boldsymbol{x}^k$, then marginalizing (7) over the range, one obtaines statistical model being the *angular Gaussian* density [19]

$$p(\hat{\boldsymbol{x}}^k|\boldsymbol{x}^{k-1}) = \frac{1}{C}\int_0^\infty s^2 \exp(-s^2/2\kappa_\tau + s\hat{\boldsymbol{x}}^{k-1}\cdot\hat{\boldsymbol{x}}^k)\mathrm{d}s \quad (8)$$

with parameters $\hat{\boldsymbol{x}}^{k-1} = \boldsymbol{x}^{k-1}/\|\boldsymbol{x}^{k-1}\|$, $\kappa_\tau = \|\boldsymbol{x}^{k-1}\|^2/\sigma_\tau^2$, and normalization constant C. Following [7], for moderate or large values of κ_τ (practically most relevant cases), (8) can be well approximated by the vMF density $f(\hat{\boldsymbol{x}}^k; \hat{\boldsymbol{x}}^{k-1}, \kappa_\tau)$.

Second, if we consider a random walk approximating the isotropic Wiener process on $\mathbb{S}^2$ [20], its transition pdf is given by

$$p(\hat{\boldsymbol{x}}^k|\hat{\boldsymbol{x}}^{k-1}) = \frac{1}{4\pi}\sum_{l=0}^{\infty}(2l+1)e^{-\sigma_\tau^2 l(l+1)}L_l(\hat{\boldsymbol{x}}^{k-1}\cdot\hat{\boldsymbol{x}}^k)\,, \quad (9)$$

where L_l are Legendre polynomials of degree l, and $\sigma_\tau > 0$ defined as above. Again, according to [16, Section 9.3.3],

$$p(\hat{\boldsymbol{x}}^k|\hat{\boldsymbol{x}}^{k-1}) \approx f(\hat{\boldsymbol{x}}^k; \hat{\boldsymbol{x}}^{k-1}, \kappa_\tau)\,,$$

where $\kappa_\tau = 1/2\sigma_\tau^2$. In further, we will assume our motion model on the unit 2–sphere being described by the von Mises-Fisher density.

Usage of physically more realistic motion models like Ornstein–Uhlenbeck or Langevin processes [21], in place of simple Wiener process, requires more complex state representation manifolds and solving the Fokker-Planck equation to obtain the corresponding state transition densities. The latter typically needs to be further approximated by an appropriate parametric density which will keep the model computationally tractable and statistically consistent with the remaining ingredients of the filtering algorithm: state distribution and measurement model.

C. Observation model

As already announced above, we assume that observation process consists of measuring object's direction, where measurement disturbances are interpreted as random rotations, i.e. observed direction $\boldsymbol{z}$ is a random rotation of the true direction $\boldsymbol{x}$. It is reasonable to statistically describe such model by a unimodal distribution which is rotationally invariant around the true direction $\boldsymbol{x}$. Our choice will be the von Mises–Fisher distribution [22], represented by its density

$$p(\boldsymbol{z}|\boldsymbol{x}) = C_3(\kappa_o)\exp(\kappa_o \boldsymbol{x}\cdot\boldsymbol{z}), \quad \boldsymbol{z},\boldsymbol{x} \in \mathbb{S}^2\,, \quad (10)$$

where the concentration parameter κ_o describes the measurement uncertainty.

D. Filtering equations

Having at hand the motion and observation models, both described by vMF densities, we can perform the simplest Bayesian tracking of a moving object. Let the object have the estimated position (direction) $\boldsymbol{\mu}^{k-1} \in \mathbb{S}^2$, conditioned upon all available measurements up to (and including) time $k-1$, which is statistically described by the density

$$p(\boldsymbol{x}^{k-1}|\boldsymbol{z}^{1:k-1}) = C_3(\kappa^{k-1})\exp(\kappa^{k-1}\boldsymbol{\mu}^{k-1}\cdot\boldsymbol{x}^{k-1})\,.$$

Predicted direction at the next time is described by the convolution of the motion model with the belief density

$$p(\boldsymbol{x}^k|\boldsymbol{z}^{1:k-1}) = \int_{\mathbb{S}^2} p(\boldsymbol{x}^k|\boldsymbol{x}^{k-1})p(\boldsymbol{x}^{k-1}|\boldsymbol{z}^{1:k-1})\mathrm{d}\boldsymbol{x}^{k-1}\,. \quad (11)$$

Since convolution of two vMF densities is not vMF, we will approximate (11) by another vMF, which is optimal choice in the sense of the Kullback-Leibler divergence [23]. Calculating the directional mean with respect to the prediction density

$$\begin{aligned}\mathbb{E}[\boldsymbol{x}^k|\boldsymbol{z}^{1:k-1}] &= \int_{\mathbb{S}^2}\boldsymbol{x}^k p(\boldsymbol{x}^k|\boldsymbol{z}^{1:k-1})\mathrm{d}\boldsymbol{x}^k \\ &= A_3(\kappa_\tau)A_3(\kappa^{k-1})\boldsymbol{\mu}^{k-1}\,,\end{aligned}$$

according to (4) we determine unique vMF $f(\boldsymbol{x}^k; \bar{\boldsymbol{\mu}}^k, \bar{\kappa}^k)$ such that $\mathbb{E}[\boldsymbol{x}^k|\boldsymbol{z}^{1:k-1}] = A_3(\bar{\kappa}^k)\bar{\boldsymbol{\mu}}^k$. Thus, the prediction equations are [7]:

$$\bar{\boldsymbol{\mu}}^k = \boldsymbol{\mu}^{k-1}\,, \qquad \bar{\kappa}^k = A_3^{-1}(A_3(\kappa_\tau)A_3(\kappa^{k-1}))\,, \quad (12)$$

where A_3^{-1} denotes the inverse to the strictly monotone function A_3 defined in (6).

Upon availability of the measurement $\boldsymbol{z}^k$ at time k, posterior density is found using the Bayes rule

$$\begin{aligned}p(\boldsymbol{x}^k|\boldsymbol{z}^{1:k}) &= \frac{p(\boldsymbol{z}^k|\boldsymbol{x}^k)p(\boldsymbol{x}^k|\boldsymbol{z}^{1:k-1})}{p(\boldsymbol{z}^k|\boldsymbol{z}^{1:k-1})} \\ &= C_3(\kappa^k)\exp(\kappa^k\boldsymbol{\mu}^k\cdot\boldsymbol{x}^k)\,,\end{aligned}$$

with corresponding update equations [7]:

$$\begin{aligned}\kappa^k &= \|\kappa_o\boldsymbol{z}^k + \bar{\kappa}^k\bar{\boldsymbol{\mu}}^k\|\,, \\ \boldsymbol{\mu}^k &= \frac{\kappa_o\boldsymbol{z}^k + \bar{\kappa}^k\bar{\boldsymbol{\mu}}^k}{\kappa^k}\,.\end{aligned} \quad (13)$$

Note that equations (12)–(13) resemble the linear Kalman filter equations for updating mean and covariance matrix of Gaussian distributions [24].

III. Tracking in Clutter on the Unit 2–Sphere

Tracking of objects in a cluttered environment requires, among other, to resolve the problem of measurement-to-object association. In this section we recall two basic probabilistic (nonbackscan) approaches, developed in seminal papers of Bar-Shalom et al. [25], [26] in the context of Poisson distributed clutter and linear models described by Gaussian distributions. Here we extend those to directional (spherical) models described by von Mises–Fisher distributions.

A. Probabilistic data association filter

First we assume a single object in track with multiple measurements where the number of false alarms is a Poisson distributed random variable. Let Z^k denote the set of all measurements that fall within the validation gate at time k

$$Z^k = \{\boldsymbol{z}_j^k \ : j = 1, \ldots, m_k\},$$

and $Z^{1:k} = \{Z^1, \ldots, Z^k\}$ the history of all measurements within the validation gate. On how the validation gate is calculated for the von Mises-Fisher distribution, please refer to [9]. We want to calculate the conditional probability density $p(\boldsymbol{x}^k|Z^{1:k})$ for all $k \geq 1$. Assume that at a given time $k-1$, the object's direction is described by the vMF density

$$p(\boldsymbol{x}^{k-1}|Z^{1:k-1}) = C_3(\kappa^{k-1}) \exp(\kappa^{k-1}\boldsymbol{\mu}^{k-1} \cdot \boldsymbol{x}^{k-1}).$$

Obtaining measurements Z^k we build the following set of hypotheses:

$$\mathcal{H}_j = \{\boldsymbol{z}_j^k \text{ is the correct measurement}\}, \quad j = 1, \ldots, m_k\,,$$

and

$$\mathcal{H}_0 = \{\text{none of the gated measurements are correct}\}\,.$$

Using the total probability formula, the posterior probability density at time k is given by

$$p(\boldsymbol{x}^k|Z^{1:k}) = \sum_{j=0}^{m_k} p(\boldsymbol{x}^k|\mathcal{H}_j, Z^{1:k}) P(\mathcal{H}_j|Z^{1:k})\,. \tag{14}$$

From the definition of $\mathcal{H}_j$ and using the Bayes rule, for $j = 1, \ldots, m_k$ we have

$$\begin{aligned} p(\boldsymbol{x}^k|\mathcal{H}_j, Z^{1:k}) &= p(\boldsymbol{x}^k|\mathcal{H}_j, Z^k, Z^{1:k-1}) \\ &= \frac{p(Z^k, \mathcal{H}_j|\boldsymbol{x}^k)p(\boldsymbol{x}^k|Z^{1:k-1})}{p(Z^k, \mathcal{H}_j|Z^{1:k-1})} \\ &= \frac{p(\boldsymbol{z}_j^k|\boldsymbol{x}^k)p(\boldsymbol{x}^k|Z^{1:k-1})}{p(\boldsymbol{z}_j^k|Z^{1:k-1})}\,. \end{aligned} \tag{15}$$

Assuming that the likelihood and the prior density are both vMF with respective parameters $\boldsymbol{z}_j^k, \kappa_o$, and $\bar{\boldsymbol{\mu}}^k, \bar{\kappa}^k$ given by (12), then the posterior density in (15) is also vMF with parameters analogous to those in (13):

$$\kappa_j^k = \|\kappa_o \boldsymbol{z}_j^k + \bar{\kappa}^k \bar{\boldsymbol{\mu}}^k\|\,, \tag{16}$$

$$\boldsymbol{\mu}_j^k = \frac{\kappa_o \boldsymbol{z}_j^k + \bar{\kappa}^k \bar{\boldsymbol{\mu}}^k}{\kappa_j^k}\,, \quad j = 1, \ldots, m_k\,. \tag{17}$$

Clearly, for $j = 0$, $p(\boldsymbol{x}^k|\mathcal{H}_0, Z^{1:k}) = p(\boldsymbol{x}^k|Z^{1:k-1})$.

Let $w_j = P(\mathcal{H}_j|Z^{1:k})$ denote the a posteriori probabilities of each feature having originated from the object in track. According to calculations in [26, Appendix]

$$w_j = \frac{p(\boldsymbol{z}_j^k|\mathcal{H}_j, Z^{1:k-1})}{b + \sum_{l=1}^{m_k} p(\boldsymbol{z}_l^k|\mathcal{H}_l, Z^{1:k-1})}\,, \quad j = 1, \ldots, m_k\,, \tag{18}$$

$$w_0 = \frac{b}{b + \sum_{l=1}^{m_k} p(\boldsymbol{z}_l^k|\mathcal{H}_l, Z^{1:k-1})}\,, \tag{19}$$

where $b = c(1 - p_G p_D)/p_D$, $c > 0$ is the clutter density, p_G is the probability that the correct feature will be inside the validation gate, and p_D is the probability that the correct feature will be detected. Density $p(\boldsymbol{z}_j^k|\mathcal{H}_j, Z^{1:k-1})$ denotes the probability density of a measurement conditioned upon past data and hypothesis that is correct, which is assumed to be known and in our case it is modeled by the vMF density

$$p(\boldsymbol{z}_j^k|\mathcal{H}_j, Z^{1:k-1}) = f(\boldsymbol{z}_j^k; \bar{\boldsymbol{\mu}}^k, \kappa_S^k)\,, \tag{20}$$

where

$$\kappa_S^k = A_3^{-1}(A_3(\kappa_o)A_3(\bar{\kappa}^k))\,. \tag{21}$$

Note that parameter κ_S^k has the role analogous to the Kalman innovation for linear models.

Having defined and calculated all the ingredients, posterior density (14) becomes a mixture of vMF densities

$$p(\boldsymbol{x}^k|Z^{1:k}) = \sum_{j=0}^{m_k} w_j f(\boldsymbol{x}^k; \boldsymbol{\mu}_j^k, \kappa_j^k)\,. \tag{22}$$

In order to estimate the object's direction $\boldsymbol{\mu}^k \in \mathbb{S}^2$, we calculate the directional mean

$$\mathbb{E}[\boldsymbol{x}^k|Z^{1:k}] = \int_{\mathbb{S}^2} \boldsymbol{x}^k p(\boldsymbol{x}^k|Z^{1:k}) \mathrm{d}\boldsymbol{x}^k = \sum_{j=0}^{m_k} w_j A_3(\kappa_j^k)\boldsymbol{\mu}_j^k,$$

and, using (4), determine the unique vMF density $f(\boldsymbol{x}^k; \boldsymbol{\mu}^k, \kappa^k)$, which is the best approximation of (22) in the sense of the Kullback–Leibler divergence by solving

$$\kappa^k = A_3^{-1}\left(\Big\|\sum_{j=0}^{m_k} w_j A_3(\kappa_j^k)\boldsymbol{\mu}_j^k\Big\|\right), \tag{23}$$

$$\boldsymbol{\mu}^k = \Big(\sum_{j=0}^{m_k} w_j A_3(\kappa_j^k)\boldsymbol{\mu}_j^k\Big)\Big/A_3(\kappa^k)\,. \tag{24}$$

The latter procedure is the analogon of computing the state estimate and covariance matrix from the mixture of Gaussians representing the posterior densities in [25], [26].

B. Joint probabilistic data association filter

Next we consider the problem of tracking several interfering objects $\{\mathcal{O}_1, \ldots, \mathcal{O}_N\}$, with the number of objects being fixed to N. The main issue is how to appropriately assign features to objects in track. In principle, PDA filter approach could be applied for each object separately, but this would implicitly assume that all measurement features originated by another object in track are Poisson distributed clutter [26], and we would like to avoid such a rough assumption.

Let $\boldsymbol{X}^k = \{\boldsymbol{x}_1^k, \dots, \boldsymbol{x}_N^k\} \subset \mathbb{S}^2$ denotes the set of object's states (directions) at time k, and assume that at a given time $k-1$ position of each object $\mathcal{O}_i$ is described by the vMF density

$$p(\boldsymbol{x}_i^{k-1}|Z^{1:k-1}) = C_3(\kappa_i^{k-1}) \exp(\kappa_i^{k-1} \boldsymbol{\mu}_i^{k-1} \cdot \boldsymbol{x}_i^{k-1}) \,.$$

Upon availability of a set of new measurements $Z^k = \{\boldsymbol{z}_j^k : j = 1, \dots, m_k\}$, the following set of hypotheses is built:

$$\mathcal{H}_{ij} = \{\boldsymbol{z}_j^k \text{ is caused by } \mathcal{O}_i\} \,, \quad j = 1, \dots, m_k \,,$$

and

$$\mathcal{H}_{i0} = \{\text{none of the measurements is caused by } \mathcal{O}_i\} \,.$$

Again, the total probability formula implies that the posterior density for object $\mathcal{O}_i$ at time k is given by

$$p(\boldsymbol{x}_i^k|Z^{1:k}) = \sum_{j=0}^{m_k} p(\boldsymbol{x}_i^k|\mathcal{H}_{ij}, Z^{1:k}) P(\mathcal{H}_{ij}|Z^{1:k}) \,, \qquad (25)$$

where densities $p(\boldsymbol{x}_i^k|\mathcal{H}_{ij}, Z^{1:k})$ are computed following the same lines and assumptions as in the previous PDA filter approach. They are vMF densities $f(\boldsymbol{x}_i^k; \boldsymbol{\mu}_{ij}^k, \kappa_{ij}^k)$ with parameters

$$\kappa_{ij}^k = \|\kappa_o \boldsymbol{z}_j^k + \bar{\kappa}_i^k \bar{\boldsymbol{\mu}}_i^k\| \,, \qquad (26)$$

$$\boldsymbol{\mu}_{ij}^k = \frac{\kappa_o \boldsymbol{z}_j^k + \bar{\kappa}_i^k \bar{\boldsymbol{\mu}}_i^k}{\kappa_{ij}^k} \,, \quad j = 1, \dots, m_k \,, \qquad (27)$$

and $\kappa_{i0}^k = \bar{\kappa}_i^k$ and $\boldsymbol{\mu}_{i0}^k = \bar{\boldsymbol{\mu}}_i^k$.

The only difference between PDA filter and JPDA filter is in calculation of a posteriori association probabilities $w_{ij} = P(\mathcal{H}_{ij}|Z^{1:k})$, where JPDA filter takes into account measurement-to-object association events jointly across the set of objects. This means that hypothesis $\mathcal{H}_{ij}$ consists of all *valid joint association events* $\mathcal{E}$ which assign feature $\boldsymbol{z}_j^k$ to object $\mathcal{O}_i$. By valid joint association events we consider those which assert that every feature lying within the validation gate region can originate from at most one object and every object can generate at most one feature. Thus, they partition the hypothesis $\mathcal{H}_{ij}$ and

$$w_{ij} = \sum_{\mathcal{E} \in \mathcal{H}_{ij}} P(\mathcal{E}|Z^{1:k}), \quad j = 1, \dots, m_k \,, \qquad (28)$$

$$w_{i0} = 1 - \sum_{j=1}^{m_k} w_{ij} \,. \qquad (29)$$

In order to compute $P(\mathcal{E}|Z^{1:k})$, two auxiliary indicator functions are introduced: *measurement association indicator* $\varphi_j(\mathcal{E})$, which indicates whether in event $\mathcal{E}$ measurement $\boldsymbol{z}_j^k$ is associated with any object, and *target detection indicator* $\delta_i(\mathcal{E})$, which indicates whether in $\mathcal{E}$ any measurement is associated with object $\mathcal{O}_i$. Following [26] and using vMF model instead of the Gaussian, we obtain

$$P(\mathcal{E}|Z^{1:k}) = B(\mathcal{E}) \prod_{\substack{l=1 \\ \varphi_l(\mathcal{E})=1}}^{m_k} f(\boldsymbol{z}_l^k; \bar{\boldsymbol{\mu}}_{i_l}^k, \kappa_{S,i_l}^k)$$

with $\kappa_{S,i_l}^k = A_3^{-1}(A_3(\kappa_o) A_3(\bar{\kappa}_{i_l}^k))$ analogous to (21), where i_l is the object index with which measurement $\boldsymbol{z}_l^k$ is associated. Next,

$$B(\mathcal{E}) = \frac{c^{\phi(\mathcal{E})}}{p_G^{\alpha(\mathcal{E})} C} \prod_{\substack{i=1 \\ \delta_i(\mathcal{E})=1}}^{N} p_D^i \prod_{\substack{i=1 \\ \delta_i(\mathcal{E})=0}}^{N} (1 - p_D^i) \,,$$

where $\phi(\mathcal{E})$ is the number of false features in joint event $\mathcal{E}$, which is assumed Poisson distributed, $\alpha(\mathcal{E}) = \sum_{j=1}^{m_k} \varphi_j(\mathcal{E})$ is the number of measurement-to-object associations in $\mathcal{E}$, p_G is the probability that the correct measurement will be inside the validation gate, p_D^i is the detection probability of object $\mathcal{O}_i$, and C is the normalization constant.

Posterior density (25) for object $\mathcal{O}_i$ is again a mixture of vMF densities, and the estimated posterior direction $\boldsymbol{\mu}_i^k$ with uncertainty κ_i^k is calculated via

$$\kappa_i^k = A_3^{-1}\left(\left\| \sum_{j=0}^{m_k} w_{ij} A_3(\kappa_{ij}^k) \boldsymbol{\mu}_{ij}^k \right\| \right) , \qquad (30)$$

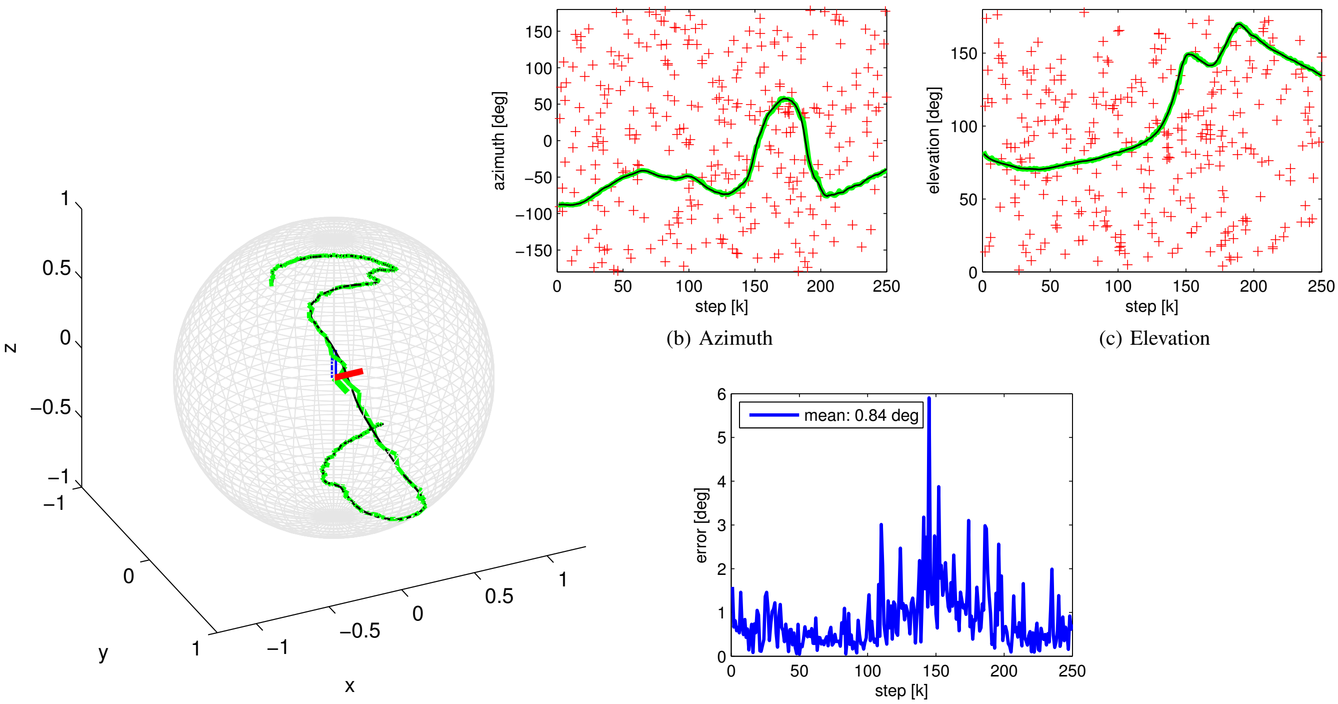
$$\boldsymbol{\mu}_i^k = \left(\sum_{j=0}^{m_k} w_{ij} A_3(\kappa_{ij}^k) \boldsymbol{\mu}_{ij}^k \right) \Big/ A_3(\kappa_i^k) \,. \qquad (31)$$

IV. Synthetic Data Experiments

In order to test the performance of the 2–sphere PDA and JPDA filter, we have simulated trajectories of a maneuvering object in 3D, where the dynamics of the system were described by a jump-state Markov model [27]. In order to make the simulations as realistic as possible (i) the trajectories were corrupted with Gaussian noise in the 3D Euclidean coordinates prior to projecting the noisy positions to the unit sphere, (ii) the probability of detection was $p_D = 0.95$ and (iii) false alarms were simulated as a Poisson process on the unit 2–sphere with the mean value $\lambda = \beta\mu(\mathbb{S}^2) = \beta \cdot 4\pi$, where μ denotes the area measure on $\mathbb{S}^2$ and the intensity β was defined as the number of measurements per solid radian. Hence, on average we could expect $4\pi\beta$ false alarms per sensor frame sampled from a uniform distribution on the unit 2–sphere. The experiments involving the PDA and the JPDA filter were envisaged so as to simulate tracking on the 2–sphere of a single moving object and multiple moving objects in clutter, respectively.

The experimental results of the tracking task involving PDA filter are shown in Fig. 2. The trajectory of the single moving object (black) and corresponding estimated trajectory (green) are shown in Fig. 2a. Azimuth and elevation for both trajectories as well as false alarms represented by red pluses are shown in Figs. 2b and 2c, respectively. The error, calculated as the great circle distance between the real and the estimated trajectories, is shown in Fig. 2d. The gained mean error value equals to $0.84°$, which confirmed the prospects of the method even with high false alarm rate.

The experimental results of the tracking task for JPDA filter are shown in Fig. 3. The trajectories of three moving objects (black) and corresponding estimated trajectories (red, green and blue) are shown in Fig. 3a. Azimuth and elevation for all the trajectories as well as false alarms represented



(a) Trajectories on the unit sphere
(b) Azimuth
(c) Elevation
(d) Error

Fig. 2: The experimental results of the tracking task for the PDA filter. The solid green line represents the estimated direction, while the solid black line is the ground truth in (a), (b), and (c). The red plus signs in (b) and (c) represent false alarms.

by red pluses are shown in Figs. 3b and 3c, respectively. The error, calculated as the great circle distance between the true and the estimated trajectories, is shown in Fig. 3d. The gained mean error values for all the tracks stayed beneath $1°$, confirming the effectiveness of the proposed method. However, the approach presented in this paper estimates only the direction of the objects without taking into account its motion model, thus not predicting the future positions of the tracked objects. This can cause, for example, that two filters either switch their tracked trajectories or both start tracking the same object. Resolving this problem will be the aim of our future research.

V. Conclusion

In the present paper we have proposed methods for tracking single and multiple moving objects in clutter, where the observation consisted of only the objects's directions. The methods are based on Bayesian tracking with an isotropic distribution on the unit 2–sphere, namely the von Mises-Fisher distribution, and the data association logic of the PDA and JPDA filters. For single object tracking we have derived the PDA filter equations by assuming a moving object in a Poisson distributed clutter. This has resulted with a mixture of hypotheses represented as von Mises-Fisher densities which were weighted by the a posteriori probability that the selected measurement is correct. For multiple object tracking the JPDA filter was derived under similar assumptions which again resulted with a mixture of hypotheses represented as von Mises-Fisher densities, where each component was weighted by the a posteriori probability of the association event. The final single component estimate for each object in track, both in the PDA and JPDA filter case, was obtained by preserving the first moment of the spherical distribution which is optimal in the Kullback-Leibler sense. In the end, the proposed methods were tested on synthetic data examples simulating a scenario of tracking a single and multiple objects in clutter on the unit 2–sphere.

Acknowledgment

This work has been partially supported by European Community's Seventh Framework Programme under Grant agreement No. 285939 (ACROSS) and from the European Regional Development Fund by the project VISTA - Computer Vision Innovations for Safe Traffic (IPA2007/HR/16IPO/001-040514). The authors would also like to thank Srećko Jurić-Kavelj for sharing his JPDA filter source code [28].

References

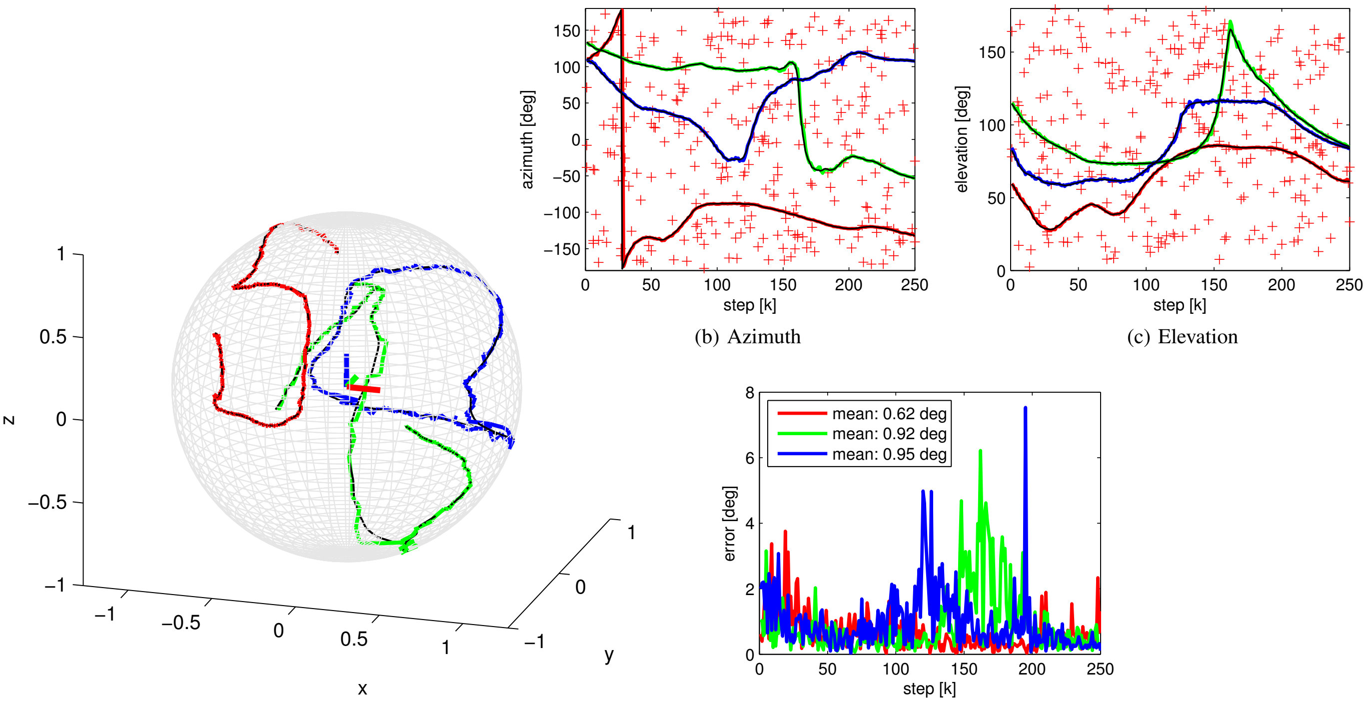
[1] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*. Artech House Publishers, 1999.

[2] Y. Bar-Shalom and E. Tse, "Sonar tracking of multiple targets using joint probabilistic data association filter," *Automatica*, vol. 11, pp. 451–460, 1975.

[3] D. Reid, "An algorithm for tracking multiple targets," *IEEE Transactions on Automatic Control*, vol. 24, no. 6, pp. 843–854, 1979.

[4] R. P. S. Mahler, "Multitarget Bayes filtering via first-order multitarget moments," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 4, pp. 1152–1178, Oct. 2003.

[5] K. Panta, D. E. Clark, and B.-N. Vo, "Data association and track management for the Gaussian mixture probability hypothesis density filter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 45, no. 3, pp. 1003–1016, 2009.



(a) Trajectories on the unit sphere

(b) Azimuth

(c) Elevation

(d) Error

Fig. 3: The experimental results of the tracking task for the JPDA filter. The solid red, green and blue lines represent the estimated directions, while the solid black lines are the ground truth in (a), (b), and (c). The red plus signs in (b) and (c) represent false alarms.

[6] B.-N. Vo and W.-K. Ma, “The Gaussian mixture probability hypothesis density filter,” *IEEE Transactions on Signal Processing*, vol. 54, no. 11, pp. 4091–4104, Nov. 2006.

[7] A. Chiuso and G. Picci, “Visual tracking of points as estimation on the unit sphere,” in *The confluence of vision and control*, ser. Lecture Notes in Control and Information Sciences, D. Kriegman, G. Hager, and A. Morse, Eds. Springer London, 1998, vol. 237, pp. 90–105.

[8] I. Marković, F. Chaumette, and I. Petrović, “Moving object detection, tracking and following using an omnidirectional camera on a mobile robot,” in *International Conference on Robotics and Automation (ICRA)*, 2014.

[9] J. Ćesić, I. Marković, and I. Petrović, “Tracking of multiple moving objects on the unit sphere using a multiple-camera system on a mobile robot,” in *Internatinal Conference on Intelligent Autonomous Systems (IAS)*, 2014.

[10] J. Glover, G. Bradski, and R. B. Rusu, “Monte Carlo pose estimation with quaternion kernels and the Bingham distribution,” in *Proceedings of Robotics: Science and Systems*, Los Angeles, CA, USA, 2011.

[11] G. Kurz, I. Gilitschenski, and U. D. Hanebeck, “Recursive monlinear filtering for angular data based on circular distributions,” in *American Control Conference (ACC)*, 2013, pp. 5439–5445.

[12] J. Glover and L. P. Kaelbling, “Tracking 3-D rotations with the quaternion Bingham filter,” Computer Science and Artificial Intelligence Laboratory, Massachusetts Institute of Technology (MIT), Tech. Rep., 2013.

[13] M. Azmani, S. Reboul, J.-B. Choquel, and M. Benjelloun, “A recursive fusion filter for angular data,” in *IEEE International Conference on Robotics and Biomimetics (ROBIO)*, Dec. 2009, pp. 882–887.

[14] I. Marković and I. Petrović, “Bearing-only tracking with a mixture of von Mises distributions,” in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2012, pp. 707–712.

[15] G. Stienne, S. Reboul, M. Azmani, J. Choquel, and M. Benjelloun, “A multi-temporal multi-sensor circular fusion filter,” *Information Fusion*, vol. 18, pp. 86–100, Jul. 2014.

[16] K. V. Mardia and P. E. Jupp, *Directional statistics*. Wiley, 1999.

[17] F. Nielsen and V. Garcia, “Statistical exponential families: A digest with flash cards,” *arXiv:0911.4863*, 2009.

[18] A. H. Jazwinski, *Stochastic processes and filtering theory*. New York: Academic press, 1970.

[19] G. S. Watson, *Statistics on Spheres*. Wiley, 1983.

[20] K. Yosida, “Brownian motion on the surface of the 3-sphere,” *Ann. Math. Statist.*, vol. 20, no. 2, pp. 292–296, 1949.

[21] F. Beichlet, *Stochastic Processes in Science, Engineering, and Finance*. Chapman & Hall, 2006.

[22] K. V. Mardia and P. E. Jupp, *Directional Statistics*. New York: Wiley, 1999.

[23] O. Schwander and F. Nielsen, “Learning mixtures by simplifying kernel density estimators,” in *Matrix Information Geometry*, F. Nielsen and R. Bhatia, Eds. Springer Berlin Heidelberg, 2013, pp. 403–426. [Online]. Available: http://link.springer.com/chapter/10.1007/978-3-642-30232-9_16

[24] S. Thrun, W. Burgard, and D. Fox, *Probabilistic Robotics*. The MIT Press, 2006.

[25] Y. Bar-Shalom and T. Edison, “Tracking in a cluttered environment with probabilistic data association,” *Automatica*, vol. 11, no. 5, pp. 451–460, 1975.

[26] T. Fortmann, Y. Bar-Shalom, and M. Scheffe, “Sonar tracking of multiple targets using joint probabilistic data association,” *IEEE Journal of Oceanic Engineering*, vol. 8, no. 3, pp. 173–184, 1983.

[27] M. Coates, “Distributed particle filters for sensor networks,” in *Information Processing in Sensor Networks*. Springer, 2004, pp. 99–107.

[28] S. Jurić-Kavelj, M. Seder, and I. Petrović, “Tracking multiple moving objects using adaptive sample-based joint probabilistic data association filter,” in *International Conference on Computational Intelligence, Robotics and Autonomous Systems (CIRAS)*, 2008, pp. 99–104.