Wavelet analysis of hydrological signals on an example of the River Sava

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Abstract - **Spectral analysis and wavelet analysis of successive flood waves on the basis of measured water levels help to understand hydrological processes and to improve hydrological modeling. In this paper, we compare Fourier transform, short time Fourier transform and continuous wavelet transforms to describe behavior of River Sava. Urban, agricultural and nature protected areas in the Republic of Croatia are under considerable influence of floods caused by the River Sava. Better understanding of the temporal patterns of the hydrological signals (discharge, water - level) is an important contribution to the water management of the River Sava.**

I. Introduction

The River Sava is one of the most important rivers for water management in Republic of Croatia. Its total length is 947 km and the river basin area covers 97,713 km^2 from witch 25,373 km^2 (26%) belongs to Croatia. With its average discharge of 1564 m^3/s, the River Sava represents the most important Danube tributary. In spring, after the snow melts, and in autumn, after the heavy rainfall, the middle part of the River Sava valley is prone to floods. Better describing and understanding of the hydrological regime is very important in the process of undertaking measures to prevent or limit flood hazards. This paper is focused on better description of water level regime in the middle reaches of the River Sava.

Natural processes and signals, such as discharge and water levels are characterized with significant departures from stationary and time–varying autocorrelation properties. These hydrological time series consist of intermittent processes where extreme events do not occur evenly. Fourier analyses have severe limitations for that kind of processes. Main shortcoming is that it identifies the frequencies present in the signal but not their moment of occurrence. Wavelet analysis gives a possible solution by time–frequency or time-scale localization of the process. An application of the wavelet analysis (WA) for the geophysical seismic signals was first introduced by Grossmann and Morlet [1]. WA founds its application in hydrology for different types of applications such as: identification of seasonal and interannual variability in temperature, precipitation and discharge [2][3]; identification of impact of climate variability on the river discharge and karst springs discharges [4][5][6]; rainfall-runoff model calibration and performance [7][8] and comparing temporal patterns of water quality [9].

In this paper, water levels from six gauging stations (GS) on the River Sava are qualitatively analyzed and compared to different spectral methods. Several advantages of the wavelet analysis are shown in this application. After wavelet decomposition, annual period is extracted and delays of the high water level between stations are obtained.

II. Signal Processing Background

A. Fourier transform, short time Fourier transform and continuous wavelet transform

Discrete time Fourier transform is given by:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}, \tag{1}$$

where $x(n)$ is a time domain signal (i.e. the observed water-level time series) and $X(e^{j\omega})$ is its frequency domain spectrum. $X(e^{j\omega})$ is periodic with the period 2π and is observed on the range $\langle -\pi, \pi]$. In order to observe time and frequency components of the signal simultaneously, short time Fourier transform is introduced:

$$X(\tau, \omega) = \sum_{n=-\infty}^{\infty} x(n)w(n-\tau)e^{-j\omega n}, \tag{2}$$

where $w(n-\tau)$ is a window function (Hann, Gaussian, rectangular,…) and τ is a time shift. The resulting two-indexed transform $X(\tau, \omega)$ is plotted in a time-frequency plane [10]. Time and frequency resolution in the time-frequency plane is constant. Due to the uncertainty principle, better localization in the time domain can be traded off for the worse localization in the frequency domain, using either longer or shorter window lengths. In some applications, the size of the window should be adjusted, i.e. wide window is better for low frequency components (provides for good frequency resolution), and shorter window is better for higher frequencies (good time resolution). For that purpose, wavelet transform is defined as follows:

$$X(\tau, a) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t)\psi^* \left(\frac{t-\tau}{a}\right) dt, \tag{3}$$

where $\frac{1}{\sqrt{|a|}}\psi\left(\frac{t-\tau}{a}\right)$ is a wavelet function, ψ^* is a complex conjugate of the wavelet function, τ is a time shift and a is a scale parameter. There are plenty different wavelet functions: Haar, Daubechies, Morlet, Shannon, Meyer,… and several of them will be considered in this paper and compared with the Fourier transform and the short time Fourier transform [10][11].

B. *Synthetic water level wave analysis*

A synthetic water-level wave is created for better understanding of obtained real world results:

$$\begin{aligned} x_1(t) &= 400 + 100 \cdot \cos(2\pi \cdot 1/T \cdot t) \\ &+150 \cdot \cos(2\pi \cdot 2/T \cdot t - T/6) \\ &+200 \cdot \cos(2\pi \cdot 12/T \cdot t)\,, \text{for } t \in \langle 0, 2T] \end{aligned} \tag{4}$$

$$\begin{aligned} x_2(t) &= 400 + 100 \cdot \cos(2\pi \cdot 1/T \cdot t) \\ &+150 \cdot \cos(2\pi \cdot 2/T \cdot t - T/6) \\ &+200 \cdot \cos(2\pi \cdot 4/T \cdot t)\,, \text{for } t \in \langle 2T, 4T] \end{aligned}$$

where $T = 365$ number of days in one year. The whole signal has the duration of four years, with half-year component and another one-year component. An extra wave with a month period is added to the first two years, and a wave with three months period to the second two years (Figure 1a). Corresponding frequency components using $\omega = 2\pi/T$ are: 0.0172 (for a year period), 0.0344 (for a half year period), 0.0689 (for three months period) and 0.2066 (for a month period).

Fourier transform is applied on the synthetic signal and all of the already mentioned frequency components are found (Figure 1b and c). However, it cannot be explicitly seen when the frequency components occurred. Therefore, the short time Fourier transform is calculated and plotted (Figure 1d). For better visibility, the interesting part is magnified and shown in Figure 1e. Obviously, short time Fourier transform is not the best possible representation for this kind of signals.

In hydrological sense, waves with long periods must be visible (one year, half year, one month), and short periods are not that important. Therefore, an analysis using wavelet transform is conducted. The wavelet transforms of the synthetic signal using several wavelet functions are compared and the results are shown on Figure 1f - 1g and are summarized in TABLE 1. Haar and Mexican hat give the least acceptable results. Most of the real wavelet functions (biorthogonal, coiflets, symlets) result in a good time resolution, except for long periods (i.e. period of one year). Complex wavelets give good frequency resolution; but still do not provide information of occurrence of events in the time domain (Figure 1h). If only real part of the transform is observed, the complex wavelets give very precise results (Figure 1i). However, real Morlet wavelet gave the best result; hence it was our choice to represent the water levels of the River Sava (Figure 1g).

TABLE 1. COMPARISON OF THE WAVELET FUNCTIONS FOR WATER-LEVEL ANALYSIS.

	time	short periods	long periods
haar	no	no	no
db3	no	yes	no
bior6.8	yes	yes	no
coif3	yes	yes	no
sym8	yes	yes	no
mexh	no	no	no
meyer	yes	yes	yes
morl	**yes**	**yes**	**yes**
cmor1-1	no	yes	yes
cmor1-2	no	yes	yes
cmor8-2	no	yes	yes
fbsp2-0.5-1	no	yes	yes
shan0.5-1	no	no	no
cgau8	no	yes	no

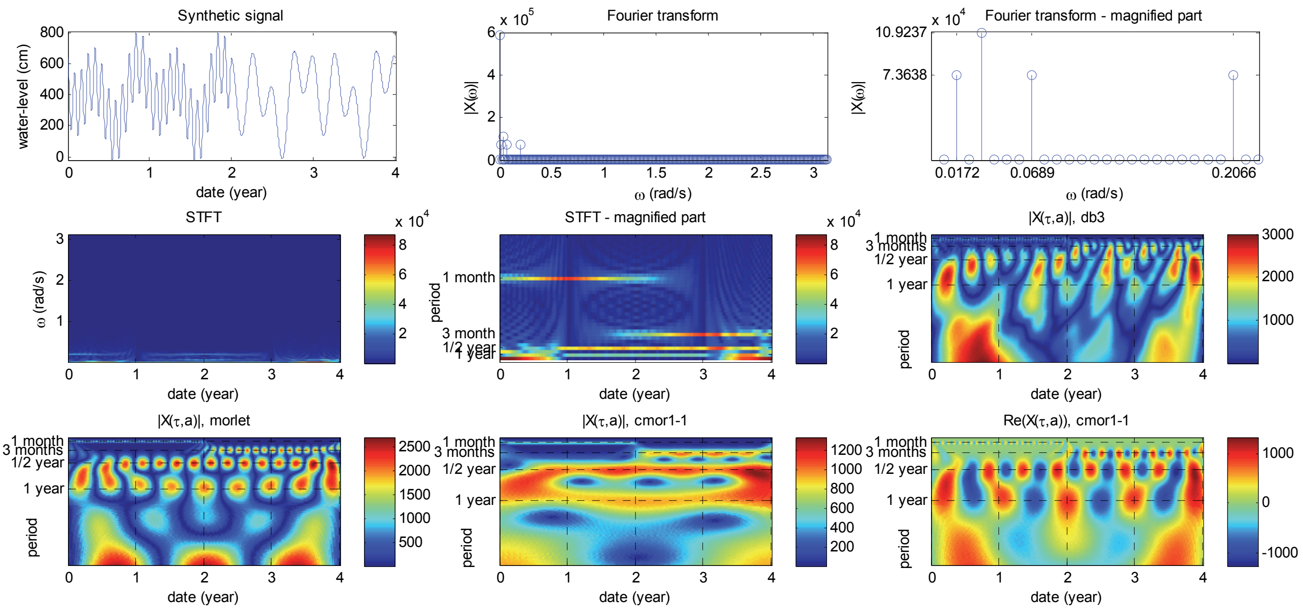

Figure 1. From left to the right and up to the down: (a) synthetic signal; (b) Fourier transform of the synthetic signal; (c) magnified part of the Fourier transform of the synthetic signal; (d) short time Fourier transform; (e) magnified part of the short time Fourier transform; (f) Daubechies3 wavelet transform; (g) absolute value of the real Morlet wavelet transform; (h) absolute value of the complex Morlet wavelet transform; (i) real part of the complex Morlet wavelet transform.

III. Analysis of the Water Level of the River Sava

The signal for the analysis was obtained as a hydrological time series of hourly water levels from six gauging stations (GS) on the River Sava (Figure 2). They are located in the middle reaches of the River Sava and the total distance between first and last GS is 184.5 km (TABLE 2) [12].

A. *Preparation of the signal*

The daily water level data were taken in the period of 25 years, between 1982 and 2007. Samples of the signal were taken every day at 2 PM. Therefore, signal consists of 9496 observations. It was checked and cleared of the possible measurement errors and was not statistically preprocessed (Figure 3). The records were incomplete on the five GS because of the war in the 1990s, which interrupted the measuring process. Data gaps were from one year on the GS Crnac, and up to ten years on the GS Gradiška (TABLE 2). Therefore, for five GS with data gaps, years 1982-1990 and 2001-2007 were considered separately.

B. *Analysis of the temporal pattern of the water levels*

First, the examination of detailed temporal pattern is conducted on GS Kobaš, using the STFT and the WT, since the data set is complete for the observed period (1982-2008). Magnified part of its STFT is shown on the Figure 4a. Longer periods are almost invisible due to pure frequency resolution and it can be clearly seen that the WT is a better representation for this type of hydrological signals (Figure 4b). The River Sava is characterized by the minimum water levels in the summer and maximum water levels in the spring and autumn. Annual scale has the highest amplitudes and its impact is the strongest. If its amplitudes are lower, the observed year is drier (1988-1991). On the other hand, if its amplitudes are higher, the year is more humid (1994). Except for annual scale impact, semiannual scale is also very strong, especially visible in dry years. When semiannual impact is lower, seasonal or monthly impacts are higher. Their influences change over the years, so the signal is evidently nonstationary.

Analysis of temporal patterns for the five remaining GS is conducted for the shorter time spans that are common for all six GS, i.e. for 1982-1990 and 2001-2008 (Figure 3). For all six GS, temporal patterns are almost identical. Information obtained using wavelet spectrogram is considered spatially consistent along all six GS (Figure 5).

This fact offers a possibility for drawing some conclusions on the missing data for the remaining five GS from the GS Kobaš data. Observed data for the remaining five GS is not stationary in time, too, because of high and low patterns in different years and seasons. For example, GS Kobaš has annual scale high water levels that occurred in years 1982-1987 and 1993-2005, but years 1988-1992 did not have expressed such annual highs. From 1995 to 1999, mentioned temporal peaks are less significant. Large variations in the wavelet transform domain are visible in years 1988-1993, and correspond to the high variation of the water levels in that period (Figure 3).

It can be concluded that there is a considerate change in stability of temporal pattern from 2001 to 2007 for almost all scales, because dominant peaks are intermittent. They are spread from seasonal scale, across semiannual and annual scales, and up to the 1.5-year and 2-year scales.

C. *Propagation of annual scales flood wave between gauging stations*

To clearly see the differences between the stations, one row from the wavelet transform for years 1982-1990 is extracted and shown in Figure 6a. The waves represent one year period. They have different amplitudes depending on the GS. Magnified parts of the waves are shown in Figure 6b and are delayed to each others. All the delays are measured and mean and median average values are calculated for years 1982-1990 and 2001-2007, separately and all together. The results are presented in TABLE 3. The average values tell how many days the annual high water level needs to come from one to another GS. It is about 3.8 days between Crnac – Jasenovac and 0.9 days between Mačkovac and Davor. As expected, the delay of flood wave propagation between gauging stations is in relation to their mutual distance. But, the annual wave shows the shortest delay between Davor and Kobaš, despite the fact they are not the closest GS. Explanation of this is in different geometrical and hydraulic characteristics of the river bed in that river section.

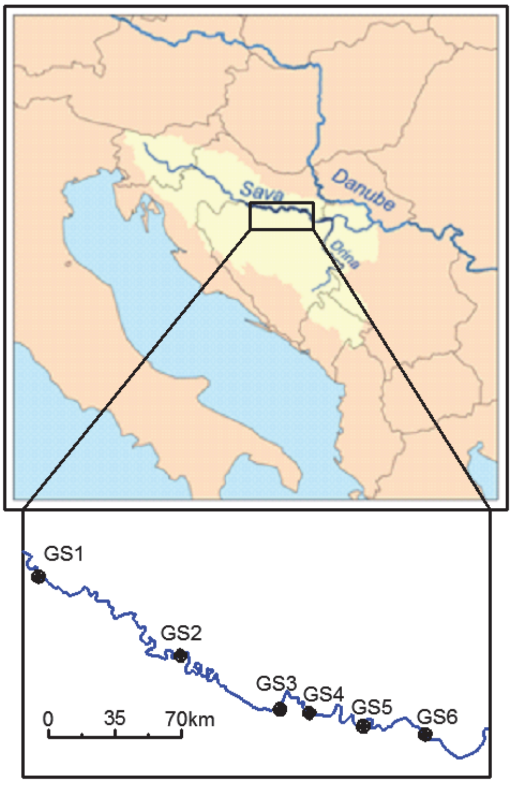

Figure 2. Sava and the analyzed gauging stations. GS1 is Crnac, GS2 is Jasenovac, GS3 is Stara Gradiška, GS4 is Mačkovac, GS5 is Davor and GS6 is Kobaš.

TABLE 2. DESCRIPTION OF THE ANALYZED GAUGING STATIONS.

	Years of missing data	Distance from river mouth [km]	Distance from previous GS [km]	GS vertical reference point [m]	Statistical characteristics [cm]			
					Mean	Std. dev.	Min	Max
GS 1 - Crnac	1993	575	74.5	91.338	115	235	-241	895
GS 2 - Jasenovac	1990-1995	500.5	47.1	82.820	256	239	-137	884
GS 3 - Gradiška	1991-2000	453.4	14.4	85.467	223	209	-106	830
GS 4 - Mačkovac	1990-1993	439	21	83.645	389	203	64	976
GS 5 - Davor	1995-1998	418	27.5	82.784	358	204	35	992
GS 6 - Kobaš	-	390.5	-	82.750	294	183	17	878

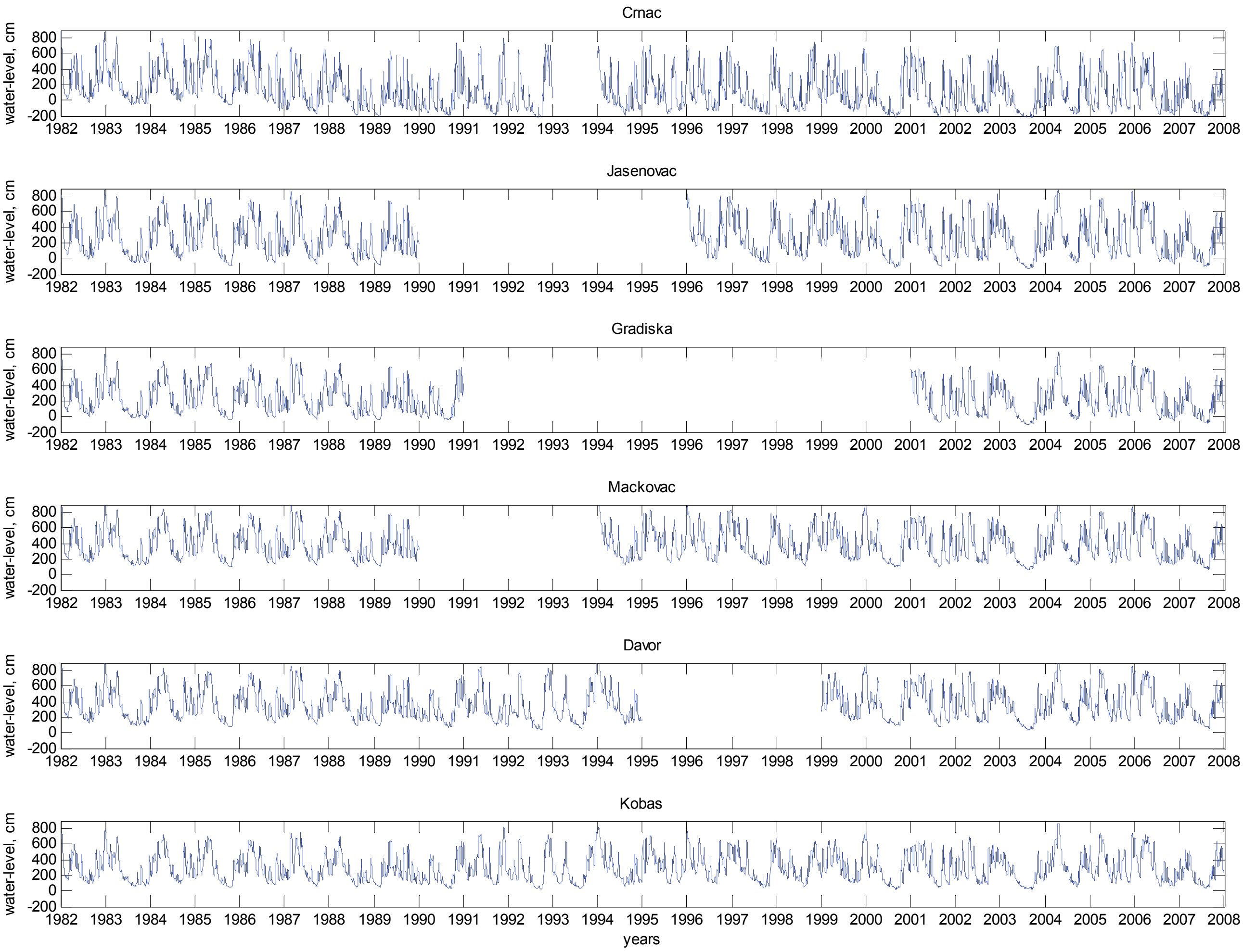

Figure 3. Water levels in time at the six gauging stations on the River Sava.

IV. CONCLUSION

The wavelet transform is a powerful tool for hydrology analysis. We obtained the best results when applying Morlet wavelet function. It gives better balance between time and frequency localization of hydrological signals than the other examined wavelet functions. Morlet wavelet was applied to water level signals from six gauging stations in the middle reaches of the River Sava. Nonstationary patterns in flood waves were examined and dominant peaks were detected: for one year, one half year, three months and one month periods. In addition, flood wave propagation time between neighboring gauging station was determined as well.

Wavelet analysis and understanding of temporal patterns in water levels has its application in improving actions for considering flood defense, e.g. early warning and conducting flood defense measures. Data used in this study is taken for the River Sava, where detailed physical hydraulic models have already existed. These hydraulic models enable precise forecasting of the flood wave amplitude and of the propagation time and velocity. Wavelet (black-box) analysis in this study is offered as an alternative to the hydraulic models. Hence, wavelet analysis could be used for other watercourses for which hydraulic models are not available.

For further research, it could be possible to define the stationary flood waves with the constant averaged water level amplitudes over all six gauging stations and for all dominant scales. This information could improve the derivation of the realistic synthetic flood waves, needed for testing of the hydraulic models of the River Sava and of the other watercourses in Croatia.

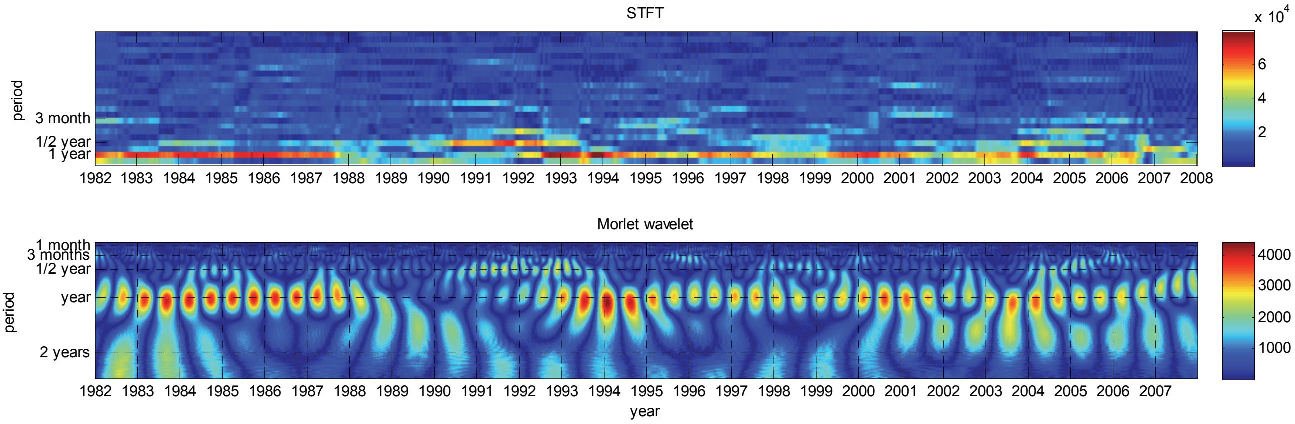

Figure 4. (a) Magnified part of the STFT for gauging station Kobaš. 1 year and 1/2 year period (scale) are clearly visible, (b) Real Morlet wavelet analysis for the same gauging station. One year period has the strongest peak, but 2 years, 1/2 year, and 3 months periods are visible, too.

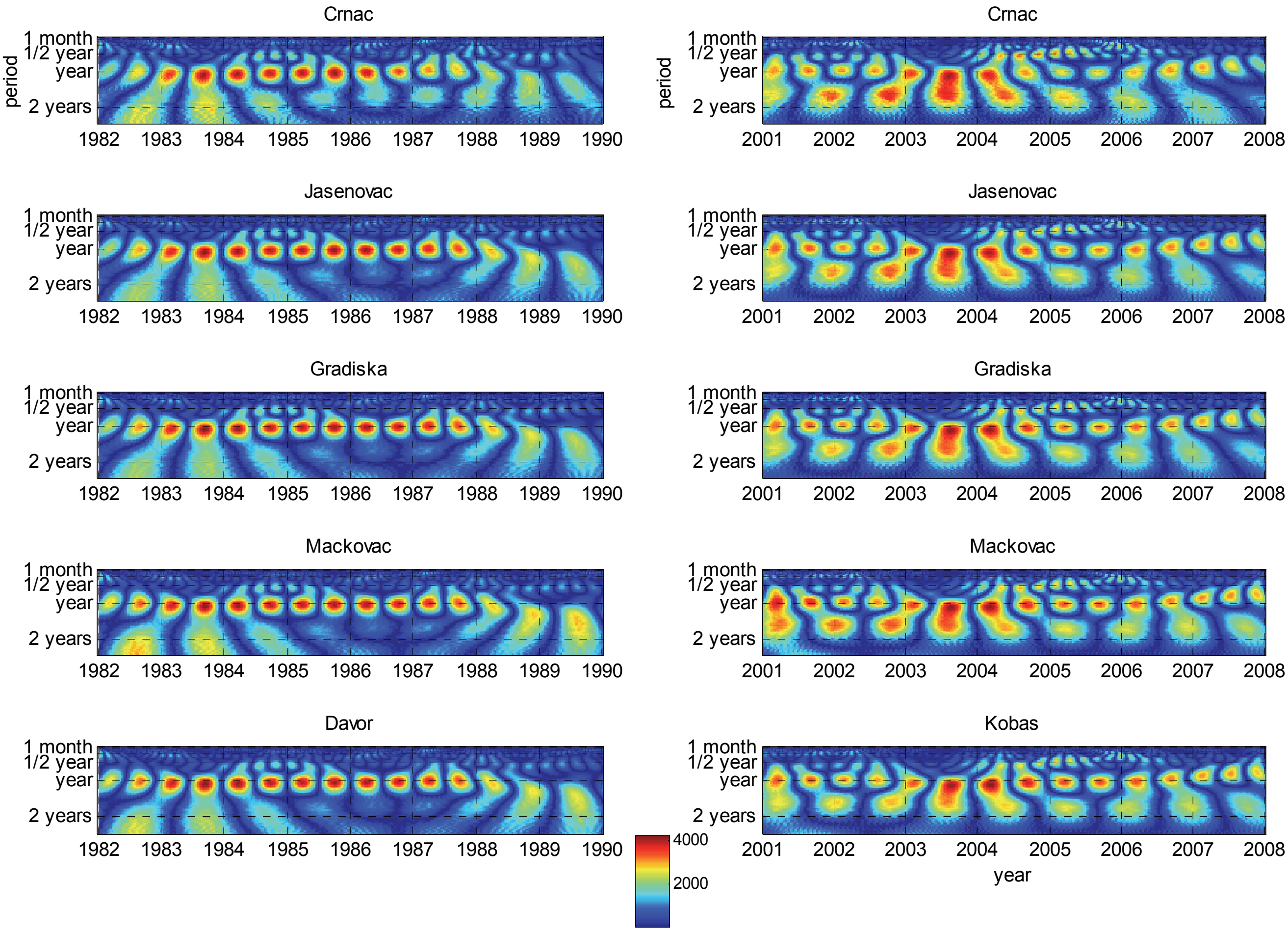

Figure 5. Apsolute value of the real Morlet wavelet analysis for the gauging stations: Crnac, Jasenovac, Stara Gradiška, Mačkovac and Davor. Because of the missing years, it is shown separately for years 1982-1990 and 2001-2008. The results in wavelet domain are similar, but with different amplitudes and delays. One year period (scale) is the strongest one, with the highest amplitudes.

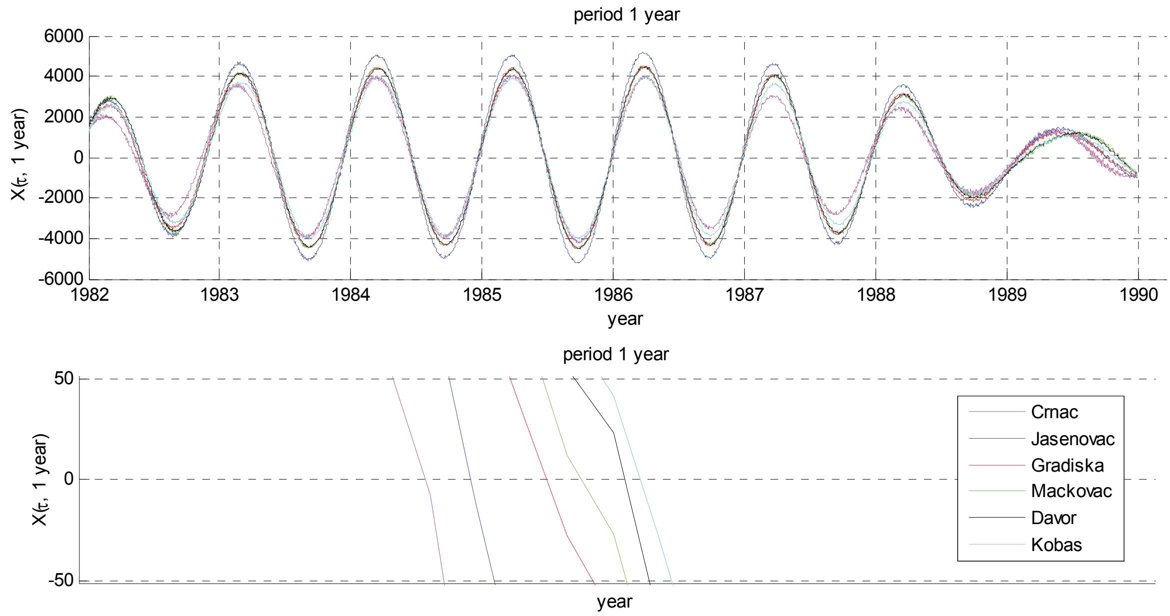

Figure 6. (a) A wave from one year period after the real Morlet wavelet transform. The waves have different amplitudes and delays when they pass through zero; (b) Magnified part of the passing through zero in the middle of year 1984.

TABLE 3. A YEAR DELAY OF THE WAVE THROUGH ALL GAUGING STATIONS.

	1982-1990		2001-2007		1982-1990, 2001-2007		Distance from previous GS, km
	mean, [days]	median, [days]	mean, [days]	median, [days]	mean, [days]	median, [days]	
Crnac – Jasenovac	2.266	2.572	5.005	6.312	3.585	3.830	74.5
Jasenovac – Gradiška	1.370	1.529	3.171	3.351	2.237	1.780	47.1
Gradiška - Mačkovac	2.710	0.926	1.104	0.160	1.937	0.764	14.4
Mačkovac – Davor	0.418	0.528	1.230	1.249	0.809	0.929	21
Davor - Kobaš	0.255	0.430	0.271	0.104	0.263	0.386	27.5

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