University of Zagreb
Faculty of Electrical Engineering and Computing
Department of Electronic Systems and Information Processing

Simple linear inversion of soil electromagnetic properties from analytical model of electromagnetic induction sensor

Darko Vasić, Davorin Ambruš, Vedran Bilas

darko.vasic@fer.hr

ACROSS
Centre of Research Excellence
for Advanced Cooperative Systems

IEEE Sensors Applications Symposium 2014

February 18 – 20, 2014

Queenstown, New Zealand

Outline

- Motivation
  - Soil properties: precision agriculture and humanitarian demining
- This paper
  - Analytical model of EMI sensor, linearization and inversion procedure
- Analytical model
  - Geometry
  - Solution
  - Usage
- Linear inversion procedure
  - Linearization
  - Inverse problem
  - Results from synthetic data
- Conclusions and future work

Motivation

- Soil electrical conductivity and magnetic susceptibility
  - Water content, salinity, clay, organic matter content, waste materials
- Upper soil layers (< 1.5 m)
  - Precision agriculture (e.g. site-specific crop management)
  - Humanitarian demining (soil effect on metal detectors with advanced classification algorithms)
- Electromagnetic induction sensors
  - Transmitter and spatially distributed receiver coils (handheld, mobile equipment)
  - Simple construction, robustness, measurement speed
- Commercial EMI sensors for upper soil layers
  - Soil apparent conductivity (uniform half-space soil model)
  - Conductivity profile trend
  - Inversion: considerable computational power and experience

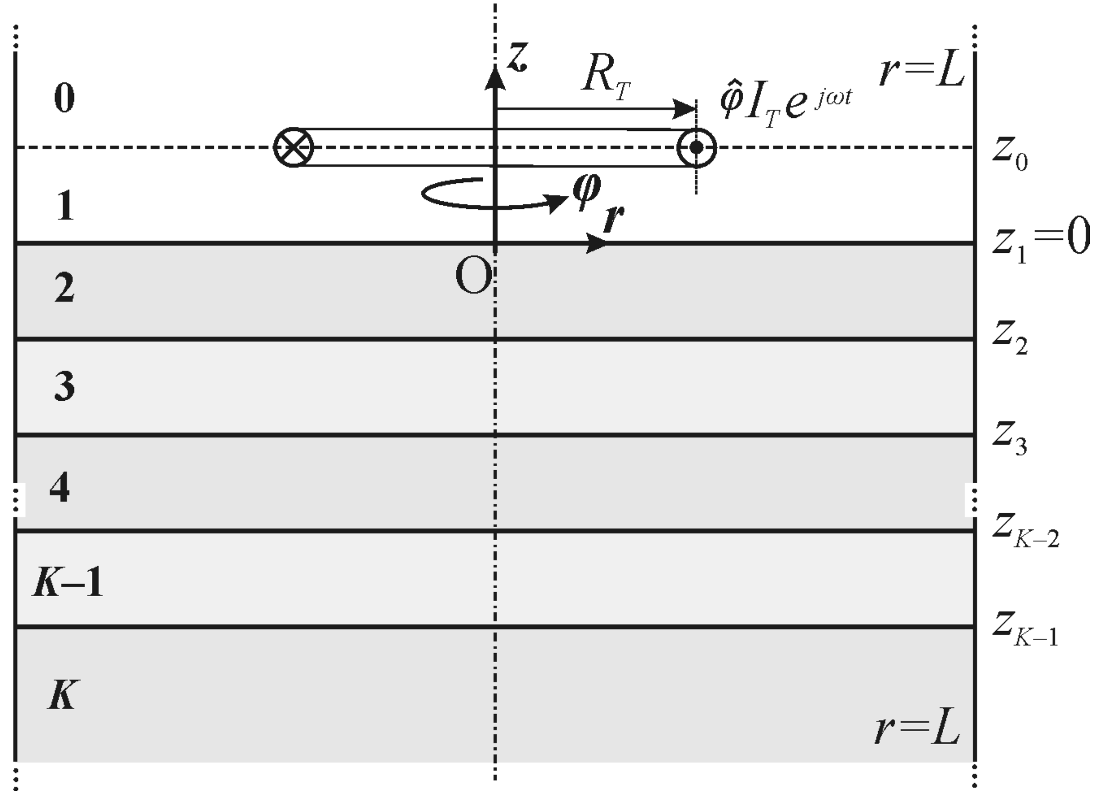
In this paper

- Analytical model of EMI sensor above arbitrary number of conductive and magnetic soil layers
  - Problem domain truncated and solution given in form of a series
  - Easier and faster implementation and error control
  - Frequency dependence of permeability is inherent
  - Time domain analysis using DFT
- Model linearization and simple linear inversion procedure
  - Least squares approach, pseudoinverse of model matrix
  - Without regularization
- Validation of the inversion procedure for selected EMI sensor configuration and synthetic data set with added measurement uncertainties

Geometry

- K-1 linear, isotropic and homogenous soil layers
- Region truncated at $r = L$
- k-th soil layer
  - Conductivity σ_k
  - Permeability μ_k
  - Thickness $\Delta z = z_{k-1} - z_k$
- Axial symmetry
- Only φ component of **A**
- Dirichlet condition:

$$A_k\,(r = L, z) = 0$$



Solution

- Magnetic potential for any region

$$A_k(r,z) = \sum_{i=1}^{N} \left[C_{k,i} \exp\left(-\alpha_{k,i} z\right) + D_{k,i} \exp\left(\alpha_{k,i} z\right)\right] J_1(\alpha_i r)$$

$$\alpha_{k,i} = \sqrt{\alpha_i^2 + j\omega\mu_0\mu_{r,k}\sigma_k}, \quad k = 0, \ldots K$$

- Eigenvalues are found solving

$$J_1(\alpha_i L) = 0, \quad \alpha_1 < \alpha_i < \alpha_n, \ i = 1, \ldots N$$

- Unknowns *C* and *D* are found using boundary conditions and orthogonal property of Bessel functions
  - *N* systems (for each *i*) of 2*K* linear equations with 2*K* unknowns

Solution

- k-th layer characteristic matrix

$$\mathbf{F}_{k,i} = \begin{bmatrix} 1 & -\frac{\mu_{r,k}}{\alpha_{k,i}} \tanh\left(-\alpha_{k,i}\Delta z_k\right) \\ -\frac{\alpha_{k,i}}{\mu_{r,k}} \tanh\left(-\alpha_{k,i}\Delta z_k\right) & 1 \end{bmatrix}$$

- Soil contribution (all layers)

$$\begin{bmatrix} g_{1,i} \\ g_{2,i} \end{bmatrix} = \begin{bmatrix} 1 & -1/\alpha_i \\ 1 & 1/\alpha_i \end{bmatrix} \prod_{k=2}^{K-1} \mathbf{F}_{k,i} \begin{bmatrix} 1 \\ \alpha_K/\mu_{r,K} \end{bmatrix}$$

Solution for region below transmitter

- Constants for region $k = 1$

$$D_{1,i} = \frac{\mu_0 R_T I_T}{L^2} \frac{J_1(\alpha_i R_T)}{\alpha_i J_0^2(\alpha_i L)} \exp(-\alpha_i z_0)$$

$$C_{1,i} = \frac{g_{1,i}}{g_{2,i}} D_{1,i}$$

$$A_1 = A_{1,\text{air}} + A_{1,\text{soil}} = \\ = \underbrace{\sum_{i=1}^{N} D_{1,i} \exp(\alpha_i z) J_1(\alpha_i r)}_{\text{Coil in air}} + \underbrace{\sum_{i=1}^{N} \frac{g_{1,i}}{g_{2,i}} D_{1,i} \exp(-\alpha_i z) J_1(\alpha_i r)}_{\text{Soil response}}$$

Usage

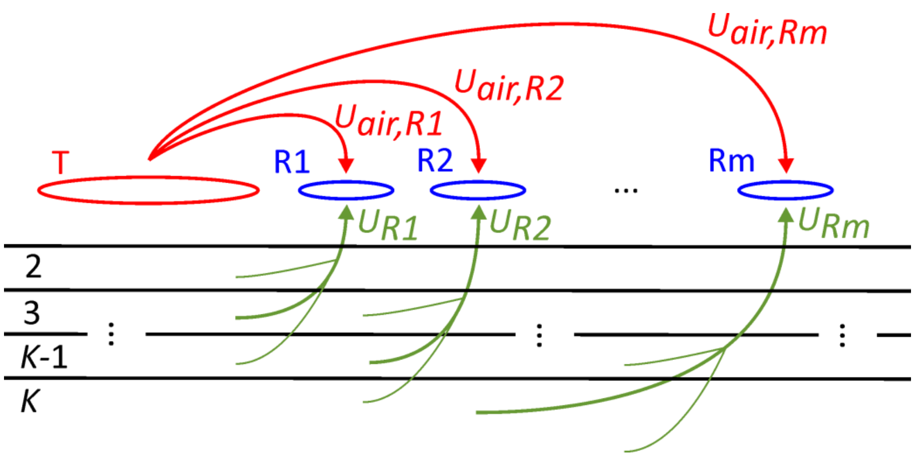
- Voltage induced in a receiver coil

$$U_R = j\omega \oint_{C_{RX}} \mathbf{A} \mathrm{d}\vec{l}$$

- Sensor head sensitivity (transmitter + one receiver coil)
- Numerical implementation
  - Number of series element increases with the size of the solution region
  - We placed region boundary at r = 5 m and used N = 20,000
  - Numerical accuracy of A is 8 digits (5 at boundaries)
  - Execution time increases linearly with N (9 ms @ N = 20,000)
- **Nonlinear in terms of magnetic susceptibility and electrical conductivity of a soil layer**

Model linearization

- Assumptions
  - Current in a layer independent of the currents in the other layers
  - Magnetic susceptibility $\chi \ll 1$
  - Inv. penetrat. depth for *k*-th layer: $\delta^2 = (\omega\mu_0\mu_{r,k}\sigma_k) / 2 \ll 1$
- Taylor expansion around $\chi = 0$ and $\delta^2 = 0$



$$\mathrm{Im}\begin{bmatrix} U_{R1} \\ \vdots \\ U_{Rm} \end{bmatrix} = \begin{bmatrix} G_{1,2}(r_{R1}) & \cdots & G_{1,K}(r_{R1}) \\ \vdots & \vdots & \vdots \\ G_{1,2}(r_{Rm}) & \cdots & G_{1,K}(r_{Rm}) \end{bmatrix} \begin{bmatrix} \chi_2 \\ \vdots \\ \chi_K \end{bmatrix} \Rightarrow \mathrm{Im}\mathbf{U} = \mathbf{G}_1\mathbf{H}$$

$$\mathrm{Re}\begin{bmatrix} U_{R1} \\ \vdots \\ U_{Rm} \end{bmatrix} = \begin{bmatrix} G_{2,2}(r_{R1}) & \cdots & G_{2,K}(r_{R1}) \\ \vdots & \vdots & \vdots \\ G_{2,2}(r_{Rm}) & \cdots & G_{2,K}(r_{Rm}) \end{bmatrix} \begin{bmatrix} \delta_2^2 \\ \vdots \\ \delta_K^2 \end{bmatrix} \Rightarrow \mathrm{Re}\mathbf{U} = \mathbf{G}_2\mathbf{\Delta}$$

Inverse problem

- Assumption
  - Soil divided into p layers of thickness Δz, except the deepest infinitely thick layer
  - Matrices G1 and G2 are of size [m x p], and vectors **H** and **Δ** of size [p x 1]
  - Single frequency and number of receivers m
- The problem is underdetermined, $m < p$
- Least squares approach, Moore-Penrose pseudoinverse

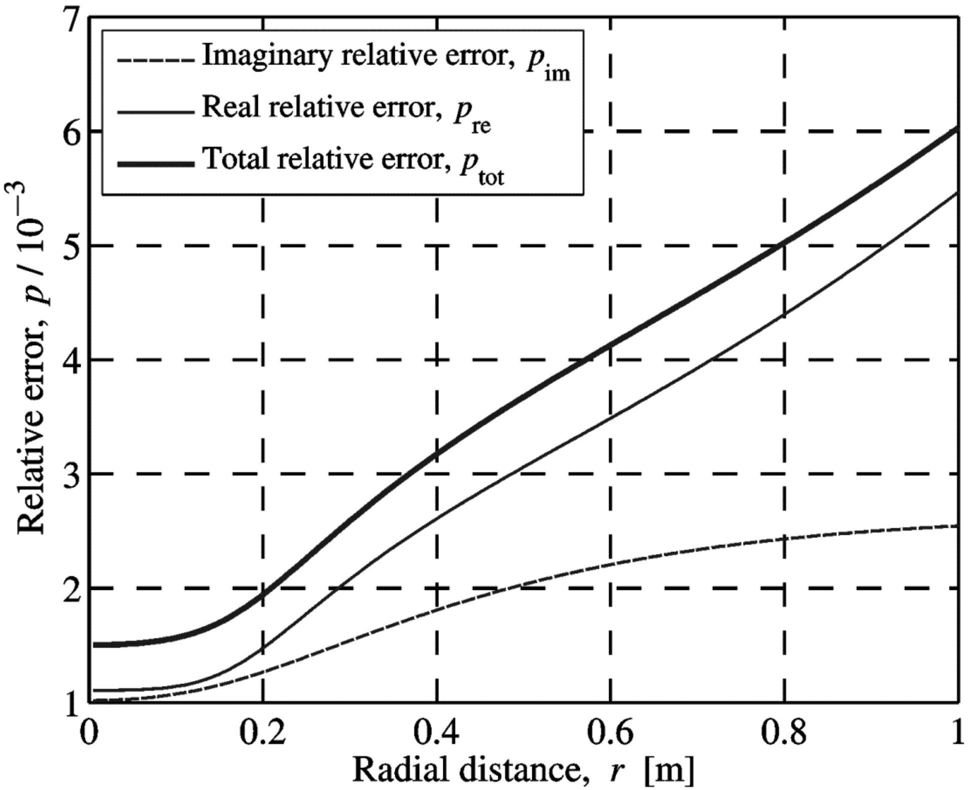
$$\mathbf{H}_{est} = \mathbf{G}_1^{\dagger} \operatorname{Im}\mathbf{U} \qquad \boldsymbol{\Delta}_{est} = \mathbf{G}_2^{\dagger} \operatorname{Re}\mathbf{U}$$

- Uncertainty analysis

$$\mathbf{C}_U = s_{rel}^2 \operatorname{diag}\left(\left|\mathbf{U}\right|\right)^2 \quad \mathbf{C}_H = \mathbf{G}_1^{\dagger}\mathbf{C}_U\mathbf{G}_1^{\dagger T} \quad \mathbf{C}_\Delta = \mathbf{G}_2^{\dagger}\mathbf{C}_U\mathbf{G}_2^{\dagger T}$$

Validation of linearization and superposition

- Example
  - Three layered soil (thickness of first two layers 10 cm)
  - Conductivity: 0.5 S/m, 0.1 S/m, 0.05 S/m
  - Susceptibility: 0.0005, 0.001, 0.005

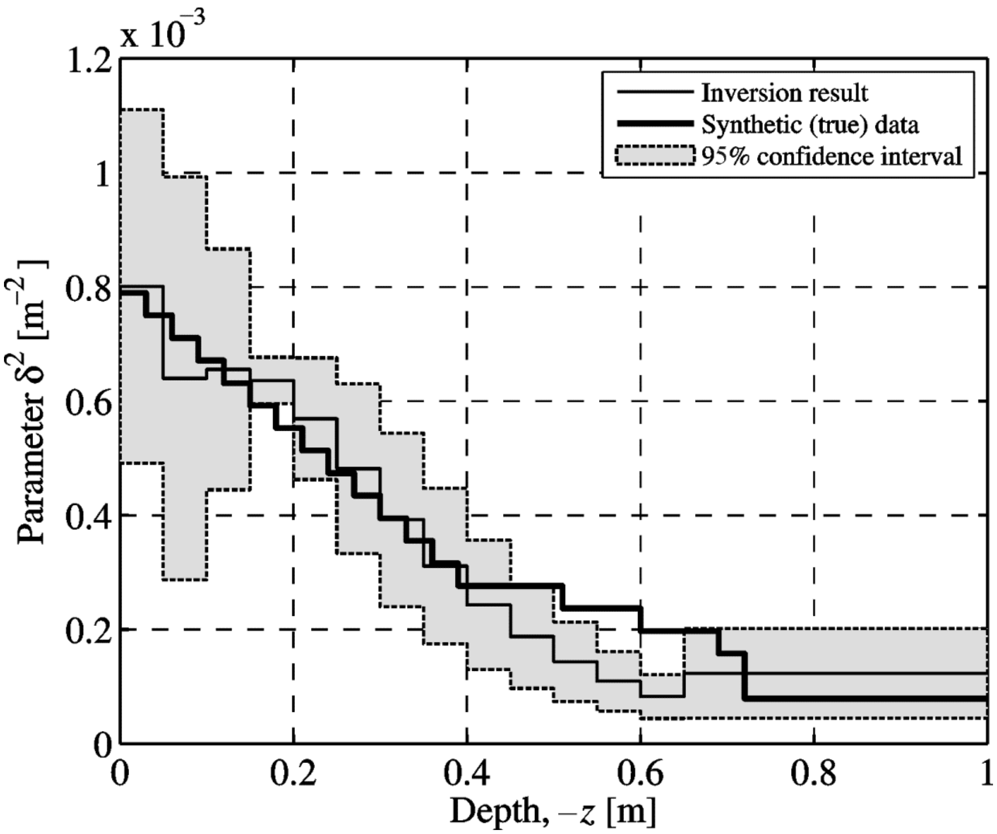


Setup of inverse problem

- Four receivers (radius 10 cm) at distances: 0 cm, 30 cm, 60 cm and 1 m
- Transmitter (radius 15 cm), single excitation frequency 10 kHz
- Coils positioned at 5 cm above the soil
- **Synthetic data**
  - 25 layers
  - Each layer (except the last) thick 3 cm
  - Calculated using the full forward model
  - Added noise (measurement uncertainty), s_{rel} = 0.5 %
  - Linear approximation error: 0.1 % – 1% (must be added to s_{rel})
- Assumed inversion soil model
  - 14 layers
  - Each layer (except the last) thick 5 cm

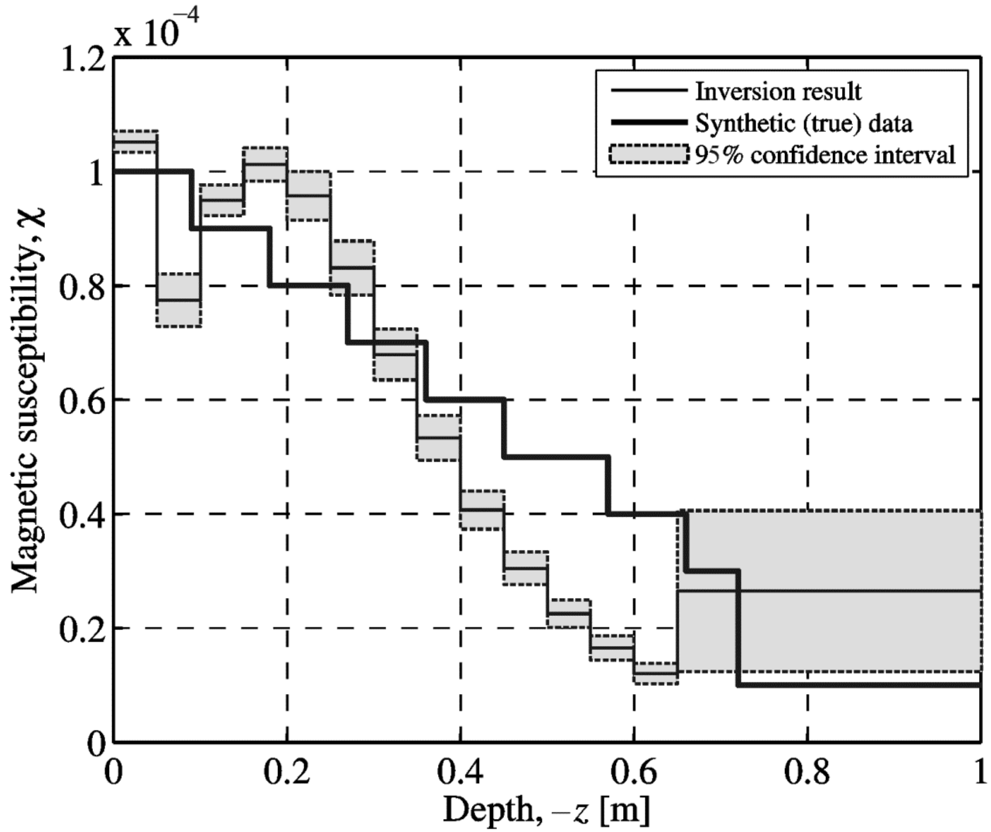
Inversion of synthetic data — δ^2

- Parameter δ^2
  - Relative error below 10% for depth down to 0.5 m (9 layers)
  - 95% confidence interval shown



Inversion of synthetic data — χ

- Magnetic susceptibility χ
  - Relative error below 20% for depth down to 0.4 m (8 layers)
  - 95% confidence interval shown



Conclusions and future work

- Forward analytical model for EMI sensor
  - Arbitrary number of soil layers, multiple receivers and frequency excitation
  - Thickness, electrical conductivity and magnetic susceptibility
- Linearized version of the model
  - For typical range of soil conductivity and susceptibility (< 1 S/m, < 0.05)
  - Fast inversion procedure → field deployable instrument
- Simple linear inversion (synthetic data)
  - Relative errors less than 20 % for the first 40 cm of the soil (for uncertainty in input data > 0.5 %)
  - Suitable for fast in-field analysis

- Future work
  - Regularization and nonlinear inversion → increased computational burden
  - Experimental verification and prototype design

Discussion

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Supported by FP7 grant agreement 285939 FP7-REGPOT-2011-1 (ACROSS)

Thank you for your attention!

darko.vasic@fer.hr