An Improvement of Incremental Conductance MPPT Algorithm for PV Systems Based on the Nelder–Mead Optimization

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***Abstract*—In this paper a maximum power point tracking (MPPT) solution for photovoltaic systems (PV) based on the Nelder–Mead (NM) optimization algorithm is presented. A well known maximum power point condition from incremental conductance (IC) algorithm is used along with the Nelder-Mead algorithm responsible for tracking maximum power point (MPP) on the PV system power curve. The presented control algorithm is able to track global maximum of the power curve under uniform insolation conditions, and more importantly under partial shading conditions with great accuracy and robustness, which was demonstrated with various test simulations in Matlab/Simulink.**

Keywords*—Maximum power point tracking, photovoltaic systems, Nelder–Mead, partial shading, uniform insolation, optimization, incremental conductance.

I. Introduction

Rapid penetration of PV systems into global electrical power production scene over the past decade caused vast engineering and scientific interest in them. On all imaginable fields, i.e. PV cell materials, power electronics, sun tracking, etc., engineers and scientists are working to improve efficiency, i.e. to maximize produced power from PV systems. Electrical power produced by PV systems is still expensive, therefore every effort towards improving system efficiency is welcome. One way to improve energy yield is to incorporate MPPT control algorithms into existing control systems of power converters.

A lot has been written in the literature about MPPT algorithms over the past years. Very first control methods were constant PV voltage or PV current, perturb and observe (P&O), hill climbing (HC), and incremental conductance (IC) algorithms. They were further extended and optimized to eliminate their disadvantages. More advanced control techniques like fuzzy logic control, sliding mode control or extremal control [1] were used in MPPT. Neural networks were utilized in this context also. A resourceful articles with detailed and comprehensive reviews of almost all today's popular MPPT methods and reported are listed in references [2]–[5].

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Today, much effort is being put into development of MPPT algorithms that are able to work during non-uniform insolation or partial shading, because all common MPPT algorithms like P&O, IC, HC and their advanced and adaptive variations work with very high efficiencies during uniform insolation. During non-uniform insolation PV power curve exhibits more maximums and some kind of global extremum seeking property must be incorporated into existing MPPT algorithms. Various algorithms based on global search methods were proposed in the literature, among the newest ones are the ones based on particle swarm optimization (PSO), genetic algorithms (GA) and differential evolution (DE) [4], [5].

In this paper we are introducing a modification of the well-known and widely accepted IC algorithm [6] based on the Nelder-Mead optimization algorithm [7]. The algorithm works accurately during uniform insolation and when partial shading occurs. Mainly all algorithms for global MPPT use global optimization technique, like DE or PSO to find global maximum when partial shading occurs and then switch to conventional P&O or IC. Our algorithm uses NM in local and global search along with the MPP condition from IC algorithm to know when it has reached MPP.

The paper is organized as follows: a generic control system for MPPT purposes is described in Section II. In Section III one-diode model of a PV panel is presented and its behavior with respect to irradiance and temperature variations under uniform isolation and partial shading is explored. In the second part of Section III convex properties of the static power-voltage (P-V) curve of the PV panel are examined, which is then used to prove Nelder-Mead algorithm convergence toward MPP in Section IV. Also, in Section IV a detailed description of the Nelder-Mead algorithm is given. In Section V MPPT algorithm based on the Nelder-Mead optimization algorithm is described in detail. Simulation tests of the proposed MPPT algorithm are presented in Section VI using real measurements of irradiance and temperature.

II. MAXIMUM POWER POINT TRACKING

In the context of MPPT in PV systems the optimization parameter could be either PV voltage, duty ratio of the converter, PV current or inductor referent current (for current mode PWM modulations). Therefore, the optimization is carried out in one-dimensional parameter space. In this paper the PV voltage will be used as an optimization parameter because under irradiance changes, which are more common and faster then temperature changes, PV voltage changes not that much as PV current. Instead of the PV voltage duty ratio can be used as well. But when the output load changes then the corresponding equilibrium duty ratio changes also, and MPPT controller has to compensate for this disturbance. In this situation it is better to have a dedicated controller for the PV voltage regulation which will adjust duty ratio or inductor referent current. Another benefit of having a dedicated controller, i.e. of choosing PV voltage as optimization variable instead of duty ratio is wider bandwidth of the control loop which allows smaller sample times of the MPPT algorithm, and as a consequence faster convergence to the MPP. The schematic of a generic control system is given in Fig. 1.

III. PV MODEL

Well-known and widely accepted one-diode model of the PV panel (see Fig. 2) will be used in this paper [8]:

$$I_{pv} = I_{ph} - I_0 \left[\exp\left(\frac{q\left(V_{pv} + I_{pv} R_s\right)}{A_k k T_c n_s} \right) - 1 \right] - \frac{U_{pv} + I_{pv} \cdot R_s}{R_p}, \tag{1}$$

$$I_{ph} = \frac{G}{G_{ref}} \left[I_{ph,ref} + \mu_{I,sc} \left(T_c - T_{c,ref} \right) \right], \tag{2}$$

$$I_0 = I_{0,ref} \left(\frac{T_c}{T_{c,ref}} \right)^3 \exp\left[\frac{qE}{kA_k} \left(\frac{1}{T_{c,ref}} - \frac{1}{T_c} \right) \right], \tag{3}$$

where $I_{ph,ref}$ is the referent photo-current, $I_{0,ref}$ is the referent diode reverse saturation current, T_c is the instantaneous PV panel thermodynamic temperature, $T_{c,ref}$ is the referent thermodynamic temperature, A_k is the diode quality factor, q is the elementary charge, k is the Boltzmann constant, E is the material bandgap energy, n_s is the number of cells in series, $\mu_{I,sc}$ is the temperature coefficient of the short-circuit current, R_s, and R_p are the equivalent series and parallel resistance, respectively, and G is the instantaneous solar irradiance. In (1) algebraic PV current-voltage dependance is implicit and this equation can only be solved numerically because it is transcendental.

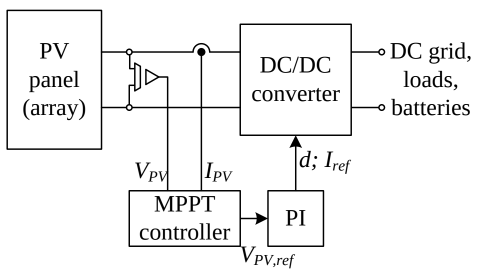


Figure 1: Conventional PV system with MPPT controller.

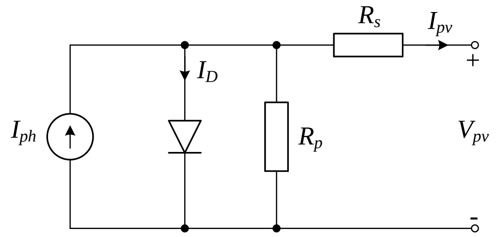


Figure 2: Electrical schematic of a PV panel.

Typical PV I-V and P-V curves are shown in Fig. 3a (in this example series string of three PV panels). Under non-uniform insolation or partial shading P-V static curve exhibits multiple peaks, and only one global maximum as shown in Fig. 3b.

Under non-uniform insolation conditions some PV panels inside series string could become power consumers, thus limiting current through series connection. This situation can even damage PV panels. To protect PV panels that are shaded external bypass diodes are connected in parallel to each of them or group of them. If on certain PV panel inside a series string irradiation drops, bypass diode shorts this PV panel or group in order to protect it from reverse polarity. When this bypassing occurs voltage of the whole string drops, and power as well. Bypass diodes are added to the series strings of PV cells inside a PV panel to protect them in the same manner.

Every stair on the static I-V curve in Fig. 3b is a consequence of bypass diode shorting associated PV panel in an array, causing multiple peaks on the power curve. The position of the global maximum can be located anywhere on the P-V curve depending on the irradiance level on each PV panel in a series string, or PV cells inside a PV panel.

A. *Convex properties of the static P-V power curve*

In order to prove the convergence of the NM algorithm convex properties of the static PV power with respect to the PV voltage curve must be analyzed. We are assuming that the PV current and voltage are always in the *working interval*, i.e.:

$$\begin{aligned} I_{PV} &\in S_I = [0, I_{sc}], \\ V_{PV} &\in S_V = [0, V_{oc}]. \end{aligned} \tag{4}$$

Short-circuit current I_{sc} and open-circuit voltage V_{oc} are temperature and irradiance dependent, but for a given fixed irradiance and temperature they are assumed to be constant.

PV power is equal to:

$$P_{PV} = V_{PV} \cdot I_{PV}. \tag{5}$$

If the PV current is considered as a function of the PV voltage $I_{PV} = f\left(V_{PV}, I_{PV}\right)$, as in (1), than the second derivative of the PV power with respect to the PV voltage is equal to:

$$\frac{d^2 P_{PV}}{dV_{PV}^2} = 2\frac{dI_{PV}}{dV_{PV}} + V_{PV}\frac{d^2 I_{PV}}{dV_{PV}^2}. \tag{6}$$

First derivative of the PV current with respect to the PV voltage can be obtained by differentiation of the first equation in (1)

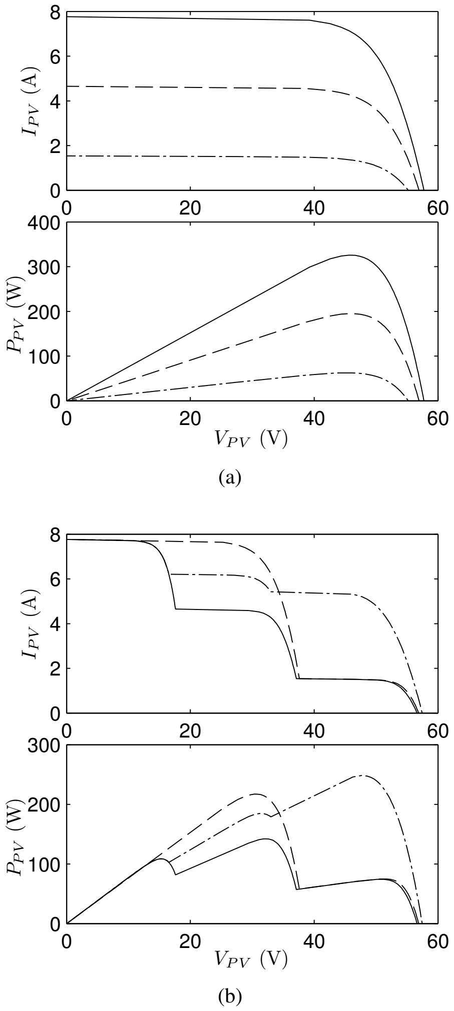


Figure 3: PV static I-V and P-V curves under: (a) uniform insolation, (b) non-uniform insolation.

with assumption that the PV current is a function of the PV voltage. It is equal to:

$$I'_{PV} := \frac{dI_{PV}}{dV_{PV}} = -\frac{R_p^{-1} + \alpha}{1 + R_s\left(\alpha + R_p^{-1}\right)}, \tag{7}$$

where:

$$\alpha = aI_0 \exp\left[a\left(V_{PV} + I_{PV}R_s\right)\right]. \tag{8}$$

Parameter a in (8) is equal to: $a = q/\left(A_k kT_c n_s\right)$. Numerator and denominator in (7) will always be positive for PV current and voltage from the interval (4). The limit of I'_{PV}, as V_{PV} approaches 0 is finite and smaller then 0. Taking all this into account we can conclude that:

$$\frac{dI_{PV}}{dV_{PV}} \leq m < 0. \tag{9}$$

Second derivative of the PV current is equal to:

$$I''_{PV} := \frac{d^2 I_{PV}}{dV_{PV}^2} = -\beta(1 + R_s I'_{PV})^2, \tag{10}$$

where $\beta = \alpha \cdot a/\left[1 + R_s\left(\alpha + R_p^{-1}\right)\right]$ is always positive. The second derivative of the PV current I''_{PV} will always be strictly negative if $I'_{PV} \neq -1/R_s, \forall V_{PV} \in S_V, I_{PV} \in S_I$. Actually this is true. If we rearrange (7), we can conclude the following:

$$I'_{PV} = -\frac{\delta}{1 + R_s\delta} = -\frac{1}{\delta^{-1} + R_s} > -\frac{1}{R_s}, \tag{11}$$

where $\delta = R_p^{-1} + \alpha$ which satisfies $\delta > 0, \forall V_{PV} \in S_V, I_{PV} \in S_I$. So we can write:

$$\frac{d^2 I_{PV}}{dV_{PV}^2} < 0. \tag{12}$$

By applying (9) and (12) on (6) we can conclude:

$$\frac{d^2 P_{PV}}{dV_{PV}^2} \leq M < 0 \tag{13}$$

In other words static power PV curve with respect to the PV voltage is strictly concave in the interval $V_{PV} \in [0, V_{oc}]$.

IV. Nelder-Mead Algorithm

Nelder-Mead optimization algorithm falls into a class of *direct search methods*. It is used as solver in unconstrained minimization of a real-valued nonlinear function $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$, where n is the number of optimization parameters and $\mathbf{x} = [x_1, x_2, ..., x_n]^T$ is the vector of parameters [7].

NM is an optimization algorithm widely used in practice for its high robustness in solving optimization tasks. There are certain problems where it could stuck in local minimum/maximum, but often it yields significant improvement in the cost function value in just a few iterations [9]. As the complexity of the optimization rises, i.e. the number of parameters, algorithm's performance deteriorates. There are few papers which cope with the convergence properties of the NM algorithm. So far the convergence has been proved for strictly convex cost functions in dimension 1 [10]. As it was explained in the previous section the static PV power curve with respect to the PV voltage is strictly concave function, so it can be concluded that NM will converge to the MPP in finite time.

NM algorithm is a simplex based optimization method. Simplex S is a set of $n+1$ points in the parameter space. Each point in the simplex is called vertex and each vertex is associated with its function value $f(v)$. As it was explained earlier, in the context of MPPT simplex is the set of two vertices, i.e. $S = \{v_1, v_2\}$. After each iteration vertices are sorted according to their associated function value in a way that the best vertex, i.e. the one with the highest function value is given index 1, and the worst one, which is associated with the lowest function value is given index n. Once the vertices are sorted so called centroid is calculated, which is equal to $c = v_1$ in the case of one-dimensional optimization problem. In the next step one or more of 5 different strategies (reflection, expansion, inside, or outside contraction, and shrinkage) are applied in order to compute a new vertex which should replace the worst vertex. In one iteration of the algorithm two function evaluations are performed.

The behavior of the algorithm is controlled by four parameters: coefficient of reflection α_r, expansion α_e, contraction α_c and shrinkage α_s.

The pseudo-code of the algorithm is given bellow:

Make initial simplex $S_0 = \{v_1, v_2\}$
Calculate associated function values $f(v_1)$ and $f(v_2)$
Sort vertices with decreasing function values
while *optimum not found* **do**
    Set the centroid as $c = v_1$
    Reflection $\rightarrow x_r = (1+\alpha_r)\cdot c - \alpha_r \cdot v_2$
    Calculate $f(x_r)$
    if $f(x_r) > f(v_1)$ **then**
        Expansion $\rightarrow x_e = (1+\alpha_r\alpha_e)\cdot c - \alpha_r\alpha_e \cdot v_2$
        Calculate $f(x_e)$
        if $f(x_e) > f(x_r)$ **then**
            Accept $x_e \rightarrow v_2 = x_e$
        else
            Accept $x_r \rightarrow v_2 = x_r$
        end if
    else
        if $f(x_r) > f(v_2)$ **then**
            Outside contraction $\rightarrow$ $x_c = (1+\alpha_r\alpha_c)\cdot c - \alpha_r\alpha_c \cdot v_2$
            Calculate $f(x_c)$
            if $f(x_c) \geq f(x_r)$ **then**
                Accept $x_c \rightarrow v_2 = x_c$
            else
                Shrink $\rightarrow v_2 = v_1 + \alpha_s(v_2 - v_1)$
                Calculate $f(v_2)$
            end if
        else
            Inside contraction $\rightarrow x_c = (1-\alpha_c)\cdot c + \alpha_c \cdot v_2$
            Calculate $f(x_c)$
            if $f(x_c) > f(v_2)$ **then**
                Accept $x_c \rightarrow v_2 = x_c$
            else
                Shrink $\rightarrow v_2 = v_1 + \alpha_s(v_2 - v_1)$
                Calculate $f(v_2)$
            end if
        end if
    end if
    Sort vertices with decreasing function values
end while

V. MPPT APPROACH BASED ON NM OPTIMIZATION ALGORITHM

The proposed MPPT algorithm has two modes of operation, *local search* and *global search*. There are two differences between them. First, different parameters of the NM algorithm are used and second, different starting points. Local search is associated with uniform insolation conditions characterized with the only one MPP on the PV power curve, while global search is initiated every time partial shading conditions are detected.

The coefficients of reflection and expansion α_r and α_e have the same value in both modes, i.e. $\alpha_r = 1$, $\alpha_e = 2$. Depending on the search mode the coefficients of contraction and shrinkage are changed. In *local* mode their values are $\alpha_c = \alpha_s = 0.5$, while in *global* mode $\alpha_c = \alpha_s = 0.9$. Larger values of α_c and α_s have a consequence that the algorithm will calculate new vertices closer to the older ones. Therefore, the size of the simplex will be changing slowly after each iteration, ensuring that the algorithm will avoid jamming in local maximums. These parameters are subject to optimization depending on actual PV array configuration, mainly how large is it. For large PV arrays they must be closer to 1, because there is a possibility of large number of peaks on the power curve. The larger value of them means slower convergence because more points are searched.

The condition to stop optimization is derived from IC algorithm, and follows directly from setting derivative of the PV power with respect to the PV voltage to zero, i.e.:

$$\begin{aligned} \frac{dP_{PV}}{dV_{PV}} &= 0 \Rightarrow I_{PV} + V_{PV}\frac{\partial I_{PV}}{\partial V_{PV}} = 0, \\ tol &= I_{PV} + V_{PV}\frac{\partial I_{PV}}{\partial V_{PV}}, \\ |tol| &\leq \varepsilon. \end{aligned} \tag{14}$$

The algorithm starts new search when $|tol| > \varepsilon$. Along with this another check is made to determine whether the search will be local or global. The search will be local if the measured PV power is changed less then predefined percentage of the measured power in the previous sample time step, otherwise the search is global. The vertices of the initial simplex are set differently for local or global search mode. The pseudo code for this part of the algorithm is:

if $|P_k - P_s| < \lambda P_s$ **then**
    local search
    $v(1) = V_{PV}[k]$
    $v(2) = 1.05v(1)$
else
    global search
    $v(1) = V_{min}$
    $v(2) = 0.8V_{max}$
end if

V_{PV}[k] is the measured PV voltage in the current time step, V_{min} and V_{max} are the minimum and the maximum input voltage of the converter, and P_s is measured PV power every m seconds. The value of m is multiple of algorithm sample time and should be set to a value of time interval in which the effects of shadowing will manifest themselves as power drop, this will usually be between 5 and 15 seconds.

VI. SIMULATION RESULTS

The PV panel used in this paper is Solvis SV36-120. The parameters of the PV panel are presented in Table I. R_s, R_p and $I_{ph,ref}$ were obtained by optimization in Matlab/Simulink using datasheet data of the PV panel. There are three bypass diodes in the PV panel so the algorithm can be tested on it without loss of generality. The boost converter used for MPPT has the following parameters: V_{in} = 8-22 V, V_{out} = 48 V, C_{out} = 880 µF, C_{in} = 360 µF, L = 69 µH and f = 100 kHz. The

Parameter	Value	Parameter	Value
$I_{ph,ref}$	7.6924 A	R_s	0.1 Ω
$I_{0,ref}$	0.236 µA	R_p	300 Ω
E	1.12 eV	$\mu_{I,sc}$	3.8 mA/°C
A_k	1.3	$T_{c,ref}$	25°C
n_S	36	G_{ref}	1000 W/m^2

Table I: Parameters of the Solvis SV36-120 one-diode model.

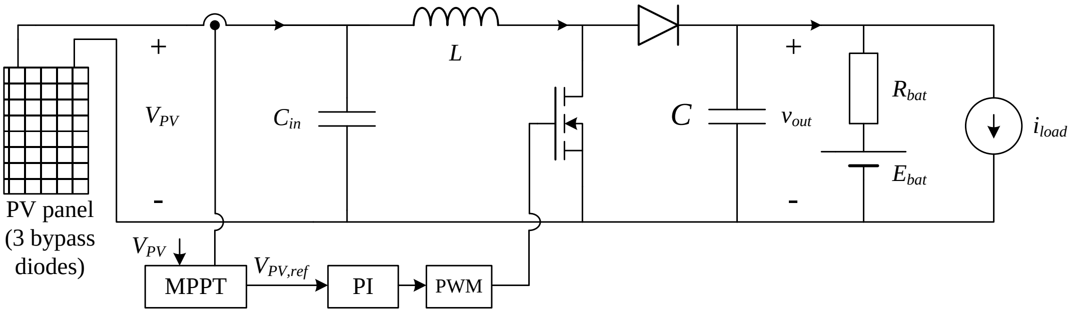
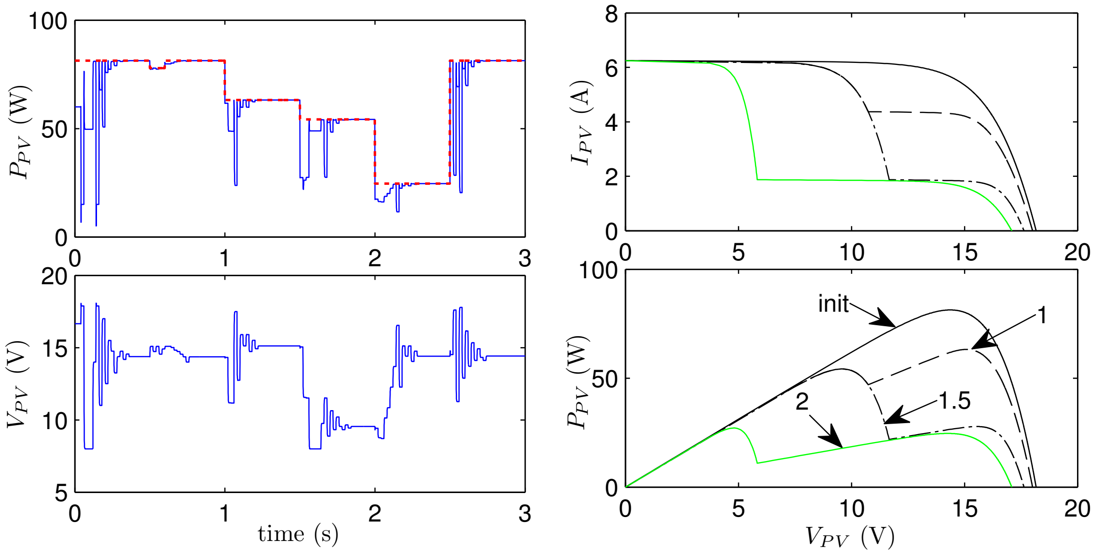


Figure 4: PV system under investigation.



(a) (b)

Figure 5: Test 1 - (a) Responses of the PV power and voltage. Maximum theoretical power is marked with dashed red line; (b) Static power curves for various conditions. Solid black - initial conditions, dashed - $t = 1$ s, dot-dashed - $t = 1.5$ s, green solid - $t = 2$ s.

schematic of a test system is shown in Fig. 4. We will perform two simulation tests.

A. *Test case 1*

This test is performed on a small time scale to show algorithm ability to find global optimum under uniform insolation conditions and under partial shading for complete system, with converter averaged model, PI controller and all relevant dynamics of the system. Responses of the PV power and voltage are shown in Fig. 5a. Sample time of the algorithm is 20 ms. Simulation is started with uniform insolation $G = 800$ W/m^2 and temperature 55°C, immediately after initialization algorithm starts global search and finds optimum after 0.3 s. At $t = 0.5$ s irradiance on one part of the PV panel is changed to $G = 720$ W/m^2 and temperature to 49.5 °C, then after 100 ms they are changed to initial values. These changes were performed to test how the algorithm is responding to slight irradiance and temperature changes. At $t = 1$ s irradiance on one section of the PV panel is changed to $G = 560$ W/m^2 which causes the appearance of two peaks on power curve. In this situation global maximum is on the right side of the power curve (Fig. 5b - dashed curve). In $t = 1.5$ s irradiance is further lowered to $G = 240$ W/m^2. This time global peak is on the left side of the power curve (Fig. 5b - dot-dashed curve). Irradiance on the second part of the PV panel is reduced from initial value to $G = 240$ W/m^2 at $t = 2$ s, which causes that the power curve has one global maximum, because of uniform insolation (Fig. 5b - green solid curve). In $t = 2.5$ s irradiances on both parts of the PV panel are raised to initial value. It can be seen that algorithm perfectly tracks global maximum of the power curve.

B. *Test case 2*

In this test simulation performance of the algorithm on a single PV panel assuming uniform insolation is examined. The used global horizontal irradiance and temperature profiles are real measurements measured on 27th April 2013 on the roof of our Laboratory for Renewable Energy Systems using Kipp&Zonen CMP11 pyranometer for global irradiance, and PT100 sensor for temperature. We have taken representative one hour sample, which contains slow and rapid changes of irradiance and temperature. The weather was partially cloudy

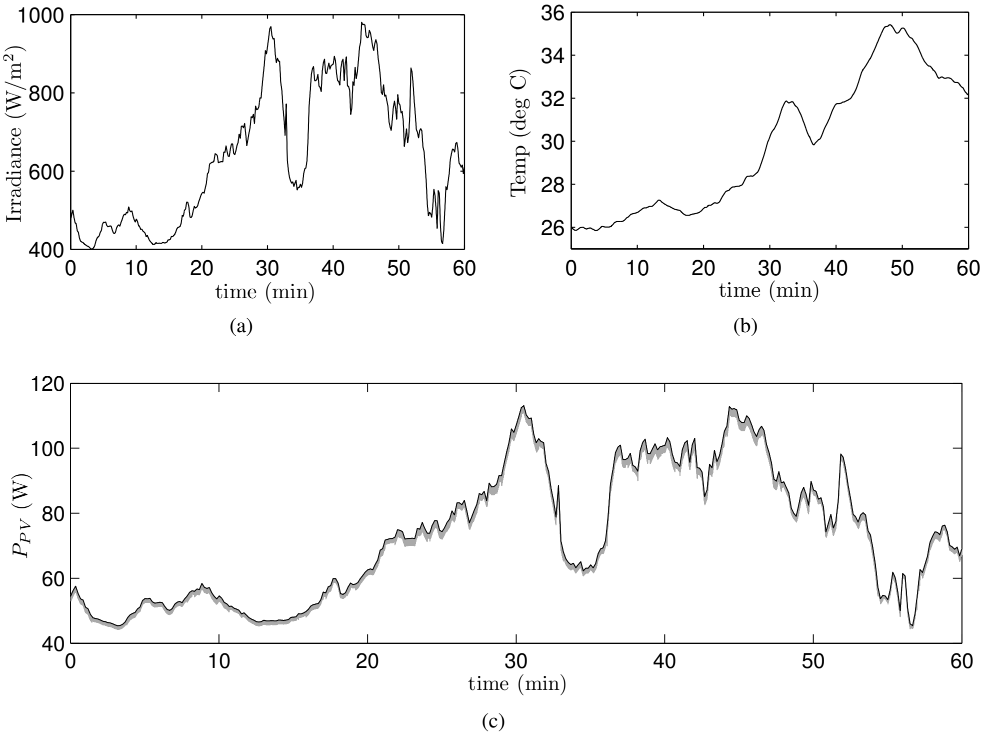


Figure 6: Test 2 - (a) Irradiance profile, (b) temperature profile and (c) extracted power in gray, and maximum theoretical power in black.

as can be seen from the irradiance profile. Irradiance and temperature profiles and extracted PV power are shown in Fig. 6. Sample time of the algorithm is 100 ms.

VII. Conclusion

In this paper we have introduced an improvement of the well-known IC MPPT algorithm by using Nelder-Mead optimization algorithm. According to the authors' knowledge, utilization of NM algorithm for MPPT purposes has not been used before in the literature. Because NM algorithm chooses potential optimum points with different distances the proposed algorithm finds optimum point faster then non-adaptive variations of IC algorithm. The performance of the algorithm under uniform insolation and partial shading was tested. It has been proved that the proposed algorithm accurately tracks global maximums under both conditions. Simulations were conducted with real measured profiles of irradiance and temperature.

In the future work we will be focusing on better partial shading determination, on experimental testing of the proposed algorithm on various PV configurations, and on experimental comparison with other MPPT solutions.

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