Nonlinear Predictive Control of a Tower Crane using Reference Shaping Approach

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***Abstract*—In this paper nonlinear Model Predictive Control of a tower crane based on reference shaping is proposed. MPC controller is used to calculate optimal reference for the inner control loop of the tower crane. The main objectives that the MPC controller needs to fulfill are tracking the reference position, suppressing the payload oscillations while satisfying operational constraints of the crane. The inner loop consists of P position controller and PI velocity controller which is common in industrial applications and easily implementable in standard frequency converters used in the cranes. The proposed approach is verified through simulation and experimental test on laboratory model of a 3D tower crane.**

***Index Terms*—3D Tower Crane, Nonlinear Predictive Control, Reference Shaping.**

I. Introduction

Crane control is a classic example of a problem where a simple feedback loop is inadequate. The hoisting mechanism of the crane is highly susceptible to oscillatory motions of the payload which can endanger both equipment and personnel. These oscillations may be triggered by both inertial forces due to the motion of the crane, and external forces such as unavoidable wind and sea conditions. Suppressing such motions usually requires a more advanced control algorithm.

Early, and still widely used, approaches for crane control used an open loop input shaping technique [1] [2]. However, they can only suppress the oscillations induced by the crane inertial forces while leaving external oscillations remain largely undamped.

Closed loop solutions for the crane problem employ a wide variety of techniques from simple PID control to advanced nonlinear approaches often involving artificial intelligence techniques. However, the most widely used approach is a combination of a linear controller and an adaptation mechanism. A linear quadratic controller (LQR) with a feedback gain vector as a function of the time-varying payload rope length is presented in [3], [4]. A robust sliding mode based approach to crane control problem in the case of poor information on system dynamics or its parameters is presented in [5]. To account for nonlinear nature of the load swinging some authors adopted fuzzy logic based approach [6], [7], adaptive fuzzy sliding mode approach [8] and passivity based control approach [9],[10]. More on existing control approaches can be found in [11].

The aim of this paper is not to replace the existing crane control system with a new one, but instead we will resort to augment it with a new position reference generation module. In order to ensure fast load manipulation that satisfies all the system and the operational constraints, we will use model predictive control (MPC) approach [13] to generate an optimal position reference signal. The proposed approach aims to combine the benefits of both, MPC and cascade control, to achieve fast reference tracking and good disturbance rejection within the inner control loops. The nonlinearity of the system is addressed via a standard linear parameter varying (LPV) framework [12] and as a result the tower crane system can be represented as a combination of three coupled subsystems. By exploiting the specific coupling structure within the 3D tower crane system the non-convex optimization problem can be transformed into three quadratic optimization problems that has to be subsequently solved.

The paper is organized as follows. In section II we present the nonlinear mathematical model of the crane. Section III discusses the details of the controller while simulated and experimental results are presented in section IV. Finally, the conclusion and future work are presented in section V.

II. Mathematical model of the Tower Crane

The tower crane, shown in Fig 1, consists of a hoisting and a trolley-jib support mechanism. The jib rotates in the horizontal plane, while trolley moves along the jib. The tower crane enables three degrees of freedom with the hoisting mechanism.

Mathematical equations of the system motion can be derived via Lagrange equations, by defining total potential and kinetic energy of the system as a functions of generalized coordinates: jib angular position θ, swing angle ϕ, trolley position x, pendulum swing angle α and rope length L (Fig 1). The resulting nonlinear model is very complex. In order to simplify the model, three motions are considered separately while couplings are treated as a change in system parameters. The obtained nonlinear model is represented by equations (1)-(4).

Using small-angle approximation a simplified mathematical model (1)-(4) can be written in the following compact form:

- hoisting system dynamics
$$x_L(k+1) = A^{(1)}x_L + B^{(1)}u_L, \tag{5}$$
where $x_L = \begin{bmatrix} L \, \dot{L} \end{bmatrix}^T$
- motion of the trolley
$$x_\alpha(k+1) = A^{(2)}(L)x_\alpha + B^{(2)}(L)u_x, \tag{6}$$

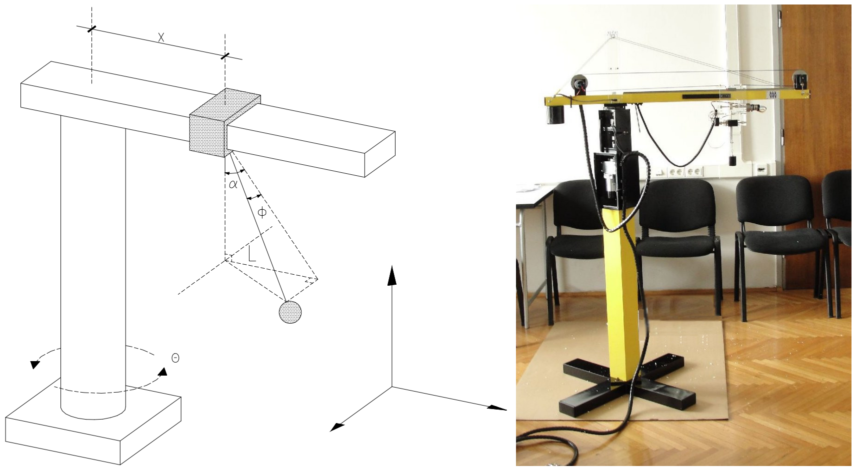

Fig. 1. Experimental model of tower crane

$$\ddot{x} = \frac{-(J_{cm}+mL^2)B_{eqx}\cdot\dot{x} + (m^2L^3 + LmJ_{cm})\sin(\alpha)\cdot\dot{\alpha}^2 + mL\cos(\alpha)B_{p\alpha}\cdot\dot{\alpha}}{(M+m)J_{cm} + mML^2 + m^2L^2\sin^2(\alpha(t))} + \\ + \frac{m^2L^2 g\cos(\alpha)\sin(\alpha) - (J_{cm}+mL^2)(\frac{\eta_{gx}K_{gx}^2\eta_{mx}K_{tx}K_m\cdot\dot{x}}{R_{mx}r_{mp}^2} + (J_{cm}+mL^2)\frac{\eta_{gx}K_{gx}\eta_{mx}K_{tx}}{R_{mx}r_{mp}}u_x)}{(M+m)J_{cm} + mML^2 + m^2L^2\sin^2(\alpha)} \tag{1}$$

$$\ddot{\alpha} = \frac{-(M+m)B_{p\alpha}\cdot\dot{\alpha} - m^2L^2\sin(\alpha)\cos(\alpha)\cdot\dot{\alpha}^2 + mL\cos(\alpha)B_{eq}\cdot\dot{x}}{(M+m)J_{cm} + mML^2 + m^2L^2\cdot\sin^2(\alpha)} + \\ + \frac{-(M+m)mgL\sin(\alpha) + mL\cos(\alpha)\frac{\eta_g K_{gx}^2\eta_m K_{tx}K_m\cdot\dot{x}}{R_{mx}r_{mp}^2} - mL\cos(\alpha)\frac{\eta_g K_{gx}\eta_m K_{tx}}{R_{mx}r_{mp}}u_x)}{(M+m)J_{cm} + mML^2 + m^2L^2\cdot\sin^2(\alpha)} \tag{2}$$

$$\ddot{\theta} = \frac{-mLr(J_{cm}+mL^2)\sin(\phi)\cdot\dot{\phi}^2 + (m^2gL^2r)\sin(\phi)\cos(\phi)}{(J_{cm}+mL^2)(J_b + (m+M)x^2 + n_gK_{g\theta}^2J_{m\theta}) - (mLr)^2\cos^2(\phi)} + \\ + \frac{-(J_{cm}+mL^2)(B_{eq\theta} + \frac{\eta_{m\theta}\eta_{g\theta}K_{t\theta}K_{g\theta}^2K_m}{R_{m\theta}})\cdot\dot{\theta} + (J_{cm}+mL^2)\frac{\eta_{m\theta}\eta_{g\theta}K_{t\theta}K_{g\theta}}{R_{m\theta}}u_\theta}{(J_{cm}+mL^2)(J_b + (m+M)x^2 + n_gK_{g\theta}^2J_{m\theta}) - (mLr)^2\cos^2(\phi)} \tag{3}$$

$$\ddot{\phi} = \frac{(J_b + (m+M)x^2 + n_gK_{g\theta}^2J_{m\theta})mgL\sin(\phi) - (mLr)^2\cos(\phi)\sin(\phi)\cdot\dot{\phi}^2}{(J_{cm}+mL^2)(J_b + (m+M)x^2 + n_gK_{g\theta}^2J_{m\theta}) - (mLr)^2\cos^2(\phi)} + \\ + \frac{-mLr(B_{eq\theta} + \frac{\eta_{m\theta}\eta_{g\theta}K_{t\theta}K_{g\theta}^2K_m}{R_{m\theta}})\cos(\phi)\cdot\dot{\theta} + mLr\frac{\eta_{m\theta}\eta_{g\theta}K_{t\theta}K_{g\theta}}{R_{m\theta}}\cos(\phi)u_\theta}{(J_{cm}+mL^2)(J_b + (m+M)x^2 + n_gK_{g\theta}^2J_{m\theta}) - (mLr)^2\cos^2(\phi)} \tag{4}$$

where $x_\alpha = [x\,\dot{x}\,\alpha\,\dot{\alpha}]^T$

- and the jib motion

$$\dot{\theta}_\phi(k+1) = A^{(3)}(x,L)\theta_\phi + B^{(3)}(x,L)u_\theta, \tag{7}$$

where $\theta_\phi = \left[\theta\,\dot{\theta}\,\phi\,\dot{\phi}\right]^T$.

Note that in model (5)-(7) the hoisting system dynamics is linear, while the trolley and the jib dynamics are described by LPV models. Cable length L is considered as a time varying parameter for trolley LPV model while trolley position and cable length constitute a time-varying parameter vector for jib LPV model. The simplified 3D crane is graphically depicted in Fig. 2.

The parameters of the 3D tower crane model are given in

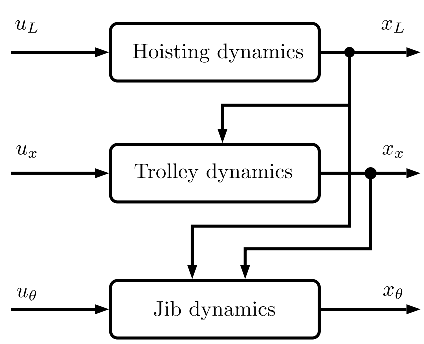

Fig. 2. Simplified 3D crane model

TABLE I
PARAMETERS OF THE TOWER CRANE

Parameters	Description
$B_{eqx} = 25[Nms/rad]$	Equivalent viscous damping coefficient as seen at the motor pinion
$B_{eqT} = 25[Nms/rad]$	Equivalent viscous damping coefficient as seen at the motor pinion
$B_{p\alpha} = 0.0015[Nm/s]$	viscous damping coefficient as seen at the pendulum axis
$B_{p\phi} = 0.0015[Nm/s]$	viscous damping coefficient as seen at the pendulum axis
$\eta_{gx} = 0.68$	Gearbox efficiency
$\eta_{mx} = 0.92$	Motor efficiency
$\eta_{gT} = 0.29$	Gearbox efficiency
$\eta_{mT} = 0.92$	Motor efficiency
$g = 9.81[m/s^2]$	Gravitational constant of earth
$J_p = 7.34 \cdot 10^{-5}[kgm^2]$	Load moment of inertia
$J_{arm} = 0.7[kgm^2]$	Arm moment of inertia
$J_{mx} = 7.32 \cdot 10^{-7}[kgm^2]$	Rotor moment of inertia
$J_{mT} = 9.44 \cdot 10^{-5}[kgm^2]$	Rotor moment of inertia
$K_{gx} = 76.64$	Planetary gearbox gear ratio
$K_{gT} = 275$	Planetary gearbox gear ratio
$K_{tx} = 0.032$	Motor torque constant
$K_{tT} = 0.02969[Nm/A]$	Motor torque constant
$R_{mx} = 25[\Omega]$	Motor armature resistance
$R_{mT} = 0.9[\Omega]$	Motor armature resistance
$M_c = 2.78[kg]$	Mass of the cart system, including the rotor inertia
$m = 0.32[kg]$	Load mass
$r_{mp} = 0.0375[m]$	Motor pinion radius

Table I.

III. MODEL PREDICTIVE CONTROL FOR REFERENCE SHAPING

Model predictive control is a form of control in which the current control action is obtained by solving, at each sampling instant, a finite horizon open-loop optimal control problem, using the current state of the plant as the initial state; the optimization yields an optimal control sequence and the first control in this sequence is applied to the plant. An important advantage of this type of control is its ability to cope with hard constraints on controls and states [13]. Specific structure of the crane system couplings (see Fig. 2) suggests possible solution to the crane control problem. Instead of solving one non-convex optimization problem we rather resort to solve it as three subsequent convex optimization problems. First MPC controller is designed for hoisting subsystem responsible for lifting and lowering the load, second for trolley and third for arm subsystems which are responsible for moving the load from point to point. The proposed solution can be summarized as follows:

1) Solve optimization problem for cable length. Based on prediction of cable length L obtain matrices $A_k^{(2)}$, $B_k^{(2)}$
2) Solve optimization problem for translation motion. Based on prediction of cable length L, and translation motion x, obtain matrices $A_k^{(3)}$, $B_k^{(3)}$
3) Solve optimisation problem for rotational motion.

As already mentioned in the introductory section, predictive controller is used in this paper generate feasible position trajectory for each of the crane subsystems (see Fig 3), such that fast load positioning is ensured, with reduced load swinging.

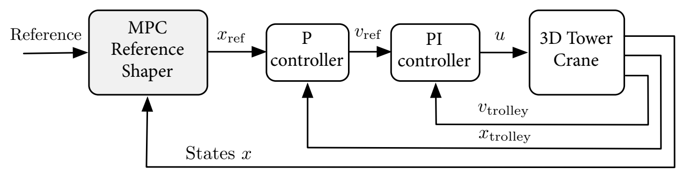

Fig. 3. Crane control loop diagram for the i-th crane subsystem

The inner loop itself consists of P position control and a PI velocity controller. Such the control structure is common in the industrial applications and usually implemented in frequency converters. Furthermore, by retaining inner loops the tower crane model becomes more robust to model uncertainties as well as external disturbance. The input to the inner loop is the position reference signal, while the output is the actuator voltage given to the tower crane. The actuator voltage can be expressed as s time invariant linear combination of states (which were given in (1)-(4))

$$u(k) = K_x x(k) + K_r x_{\text{ref}}(k) \tag{8}$$

where K_x and K_{ref} are vectors that depend on the parameters of the P and PI controller.

The closed loop system matrix $A_{cl}^{(i)}(x^{(j)})$, $j < i$ of the i-th tower crane subsystem model can be written as

$$A_{cl}^{(i)}(x^{(j)}) = A^{(i)}(x^{(j)}) + B^{(i)}(x^{(j)})K_x^{(i)}, \tag{9}$$

where $A^{(i)}(x^{(j)})$ and $B^{(i)}(x^{(j)})$ describe the motion of the i-th crane subsystem. The input signal to the inner control loop, at time instance k, becomes the reference position $x_{ref}^{(i)}(k+j)$, $j = 1, ..., N$ (which is different from the global reference position given to the MPC). In the way, the proposed approach can be also seen as a special case of reference shaping approaches [2]. Closed loop dynamics of the i-th crane subsystem can be written as

$$x^{(i)}(k+1) = A_{cl}^{(i)}(x^{(j)})x^{(i)}(k) + B_{cl}^{(i)}(x^{(j)})x_{ref}^{(i)}(k),\ j < i. \tag{10}$$

Without loss of generality, the position tracking problem can be reformulated into a regulation problem, by translating the origin of the system, with global reference signal assumed to be zero.

In order to design MPC reference shaper in this paper we propose using the following objective function:

$$\begin{aligned} J_k^{(i)}(x_k^{(i)}, u_k^{(i)}) = \sum_{j=0}^{N} &(x^{(i)}(k+j))^T(\rho^k Q^{(i)})x^{(i)}(k+j) + \\ &+ (u^{(i)}(k+j))^T(\rho^k R^{(i)})u^{(i)}(k+j) + \\ &+ (x_{\text{ref}}^{(i)}(k+j))^T Q_{ref}(x_{\text{ref}}^{(i)}(k+j)), \end{aligned} \tag{11}$$

where $\rho \in [0, 1\rangle$, and $Q \geq 0$ and $R \geq 0$, $Q_{ref} > 0$. The idea behind such the the objective function will become apparent later in this section. In that case the finite time optimal control problem for reference shaper can be rewritten as

$$(x^{\star(i)}, x_{\text{ref}}^{\star(i)}) = \arg\min J_k^{(i)}(x^{(i)}(k), x_{\text{ref}}^{(i)}(k)) \tag{12}$$

subject to:

$$x^{(i)}(k+1) = A_{cl}^{(i)}(x^{\star(j)})x^{(i)}(k) + B_{cl}^{(i)}(x^{\star(j)})x_{ref}^{(i)}(k),\ j < i, \tag{13}$$

$$u^{(i)}(k) = K_x^{(i)}x^{(i)}(k) + K_r^{(i)}x_{\text{ref}}^{(i)}(k), \tag{14}$$

$$u_{\min} \leq K_x^{(i)}x^{(i)}(k) + K_r^{(i)}x_{\text{ref}}^{(i)}(k) \leq u_{\max}. \tag{15}$$

Under a mild assumption that the inner loop of each subsystem ensures steady-state position tracking, possibly with an oscillatory response, the stability of the proposed control scheme can be easily shown. Since $\lim_{k\to\infty}\rho^k = 0$, the objective function becomes

$$J_{k\to\infty}^{(i)} = \sum_{j=0}^{N}(x_{\text{ref}}^{(i)}(k+j))^T Q_{ref} x_{\text{ref}}^{(i)}(k+j), \tag{16}$$

which penalizes the distance of the reference signal $x_{\text{ref}}^{(i)}$ from the origin. Since there always exist reference signal to the inner loop which is closer to the origin, such that constraint satisfaction is guaranteed, it is obvious that $x_{ref}(k) \to 0$ as $k \to \infty$ is always feasible reference. Furthermore, it is minimizer of objective function $J_{k\to\infty}$ when no constraints are active. Therefore $x_{ref}(k) \to 0$, as $k \to \infty$. Since inner loop ensures the reference tracking, we have $x(k) \to 0$ as $k \to \infty$.

Tuning parameter ρ in objective function (11) can be seen as trade off between suppression of oscillation and speed of convergence towards the origin. In other words, with $\rho = 0$, the MPC reference shaper force the reference x_{ref} signal to zero as fast as possible, subject to input constraints, while with $\rho = 1$ the MPC is allowed to fully determine the optimal reference signal for the inner loop. Since both, inner controller and the model, which is assumed to be disturbance free, are known to the MPC, it will try to cancel out the action of the inner loop controller, and impose its own optimal control action, leading to steady state error in the presence of disturbance, since there is no integral action in the MPC controller. By choosing $0 < \rho < 1$, the oscillation suppression is enabled in the beginning of transient response, while later towards the end of the transient response the disturbance rejection in the inner loop become active, forcing the system convergence to the origin.

In order to enable a fast execution of the MPC law in this paper we have used an interior point based tailored solver for optimization problems, generated using online code generation tool Forces[14].

IV. Simulation and Experimental Results

MPC control of LPV model of tower crane with reference shaping was tested on the laboratory model of the 3D tower crane described in section II, for the trolley and hoisting subsystem. The optimization problems were formed as described in section III. An interior point based tailored solver for optimization problems were generated using online code generation tool Forces[14]. Simulation and experimental results of proposed controller, for simultaneous hoisting and trolley movement, with prediction horizon $N = 20$ and sample time $T_s = 0.1\,\text{s}$ are shown in Fig. 4. The system has been tested for the trolley position reference $x_{ref} = 0.2\,\text{m}$ and rope length change from $L = 0.3\,\text{m}$ to $L = 0.8\,\text{m}$. Motor voltage were constrained to lie within boundaries $u \in [-12V, 12V]$. Simulation results show fast and precise positioning and reduction of load swinging (less than $3°$). Experimental results show very good tracking of the reference position with slight overshoot which is result of discrepancy between real system and its mathematical model. Besides the proposed control approach is real-time feasible since solving both optimization problems took on average less than 2 ms on Intel Q6600@2.4 GHz.

V. Conclusion

In this paper a model predictive control of an LPV model of 3D tower crane with reference shaping is proposed. Instead of replacing the existing crane controller we used MPC to modify the position reference signal to ensure the optimal crane behavior, while satisfying the system and operational constraints. Additionally, the proposed approach allows the user to make a trade-off between suppression of load oscillations and speed of the system convergence towards the origin, by proper tuning of the scalar coefficient ρ. The coupling problems between the trolley and the hoisting subsystems are successfully solved by the proposed method. The results show that MPC reference shaping in combination with the existing controllers ensure a fast and accurate load positioning and prevents a significant load swinging. Using a tailored solver generated specifically for our problem makes the proposed control approach real-time feasible, even when stringent time constraints are used.

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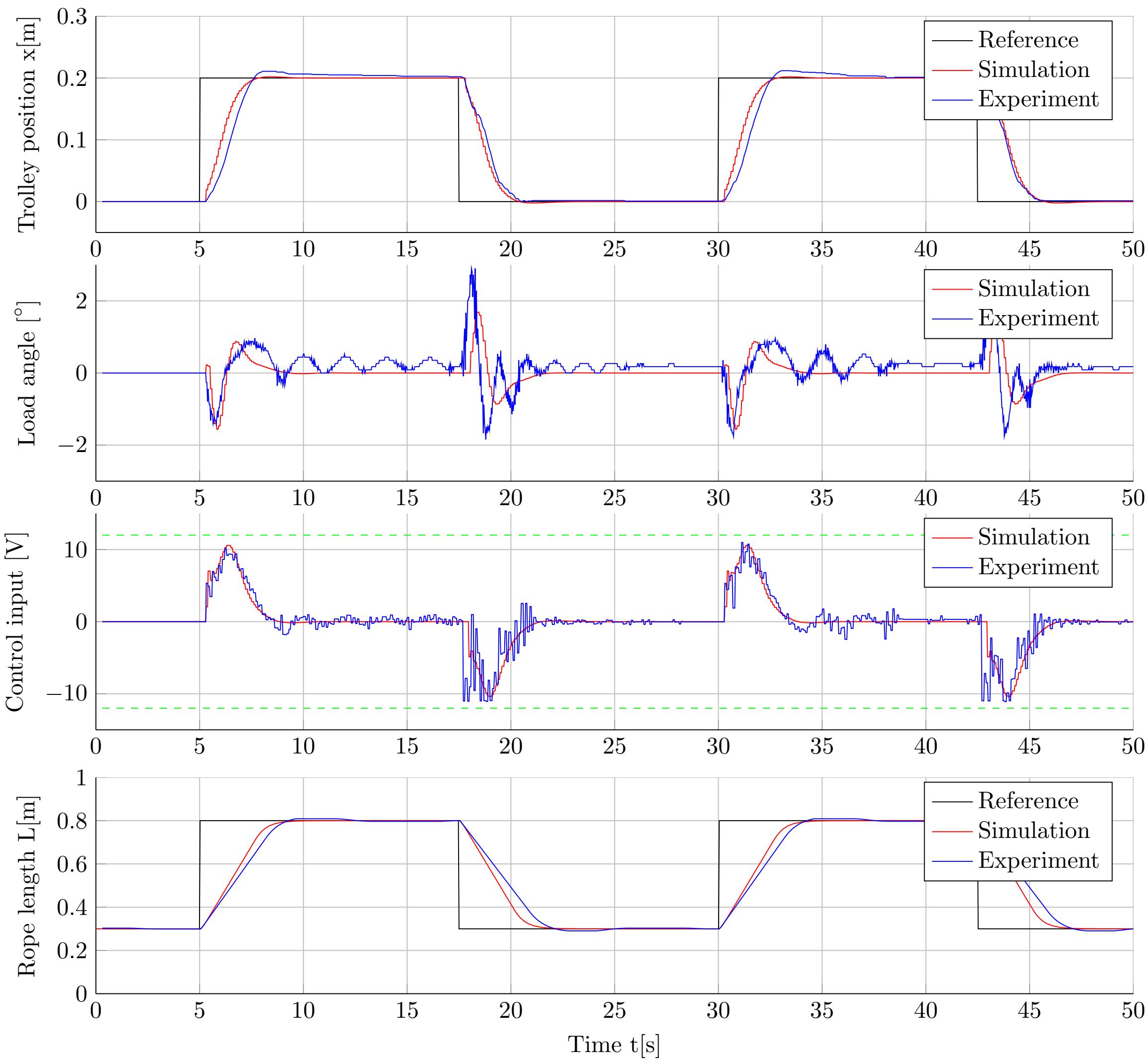

Fig. 4. Comparison of simulated and experimental obtained measurements when the rope length is oscillating between 30 and 80 cm, while the reference position is oscillating between 0 and 20 cm.

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