Detecting the Number of Components of Multicomponent Nonstationary Signals Using the Rényi Entropy of Their Time-Frequency Distributions

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Abstract In this paper, the Rényi entropy has been applied to different time-frequency distributions (TFDs) (namely, the Wigner-Ville distribution, the Choi-Williams distribution, and the spectrogram), with the purpose of determining which of the mentioned TFDs have best properties for estimation of the number of components when there is no a priori knowledge of the signal. The optimal parameter of the Rényi entropy, α, for each TFD is determined. Simulations show that the Wigner-Ville distribution has best properties among the selected TFDs since it is invariant to time duration and frequency bandwidth variations of the analysed signal. The concept of a *class*, when the Rényi entropy is applied to TFDs, is also introduced.

Introduction

Time-frequency distributions (TFDs) show nonstationary signals simultaneously in the time and frequency domain, indicating the presence of individual components. A fundamental tool in measuring the information content of a given probability distribution is the entropy function. The marginal properties exhibited by quadratic TFDs [2] are the same as those of probability density functions, therefore the generalized Rényi entropy (RE) can be applied to quadratic TFDs as a tool for detection of the number of components in the signal [1,3,4].

Information measures, as they have been used in probability theory, have been also applied to the time-frequency (TF) plane in [3], by treating TFDs, $\rho_z(t, f)$, as density functions [5].
The α^{th} order RE of the normalized TFD is defined as [2,3]:

$$R_\alpha = \frac{1}{1-\alpha}\log_2 \iint \left(\frac{\rho_z(t, f)}{\iint \rho_z(u,v)\, du\, dv} \right)^\alpha dt\, df \tag{1}$$

The RE is well-defined as long as the integral in Eq. (1) is greater than zero [3]. As α approaches 1, Eq. (1) approaches the Shannon entropy function [3] (due to the logarithm in the definition, $\alpha = 1$ is not recommended for time-frequency distributions with negative values). Since the cross-terms oscillatory structure cancels under the integration with odd powers, α should be an odd integer value for a general quadratic TFD [2], [3].
An important property of the RE is its ability to count the number of signal components in a TFD [3]:

$$R_\alpha(A_n) - R_\alpha(A_1) = \log_2 n \tag{2}$$

where $R_\alpha(A_1)$ is the entropy value of one component of the signal, and $R_\alpha(A_n)$ is the entropy value of *n* components. In the case of a two component signal, Eq. (2) becomes:

$$R_\alpha(A_2) - R_\alpha(A_1) = \log_2 2 = 1. \tag{3}$$

Note that shifting the signal in the TF plane doesn't affect the value of its RE [3].

Components separation in time and the RE

The effects of different choices of the parameter α have been tested on a signal whose components present variable distances in time. The components are two Gabor logons whose time distance increases from 0 to 100 *s*. It has been shown in [3] that, for a quadratic TFD, if the first component is supported on the time interval [0, ε], and the second one is supported on [Δt, $\varepsilon + \Delta t$], then

$$\iint X^{\alpha}{}_{1,2}(t, f)\, dt\, df = 0 \tag{4}$$

holds when

$$\Delta t > \frac{1}{2}(\alpha + 1)\varepsilon \tag{5}$$

for odd α and if the components are located on the same frequency ($\Delta f = 0$), where $X^{\alpha}{}_{1,2}(t, f)$ is the TFD of the cross-component, and Δt is the time separation between the components. This means that for odd α, the oscillatory structure of $X^{\alpha}{}_{1,2}(t, f)$ cancels under the integration for Δt sufficiently large. For $\alpha = 3$ the right side of Eq. (5) has minimal value, therefore Eq. (3) holds for close components in time.
Fig. 1 shows the Wigner-Ville distribution (WVD) of two Gabor logons with increasing time separation, while the RE for different values of α w. r. t. the time distance between the Gabor logons is shown in Fig. 2. Note that for even α Eq. (5) is not valid, because the cross-components are not annulated in the integration.

For $\Delta t < 10$ only one component is detected. As the separation increases, the components become well-defined, as shown by the exponential rise of the RE. For $\Delta t \in \left[50, \frac{1}{2}(\alpha + 1)\,\varepsilon \right]$ the maximum entropy starts to oscillate; the oscillation are reducing as Δt approaches $\frac{1}{2}(\alpha+1)\,\varepsilon$. For $\Delta t \in \left[\frac{1}{2}(\alpha+1)\,\varepsilon, \infty\right]$ the oscillations stop, and the components are completely separated. For R_3 the oscillation are minimal and the settling time is shorter.

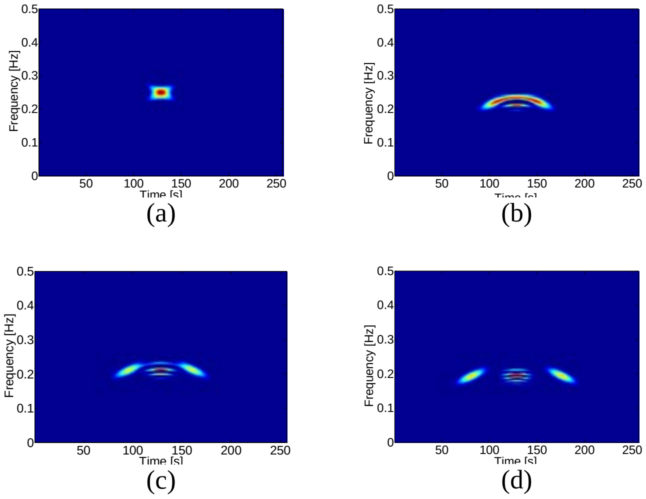

Figure 1. Separation of two Gabor logons in the TF plane: $\Delta t = 0$ s (a), $\Delta t = 35$ s (b), $\Delta t = 65$ s (c), $\Delta t = 95$ s (d).

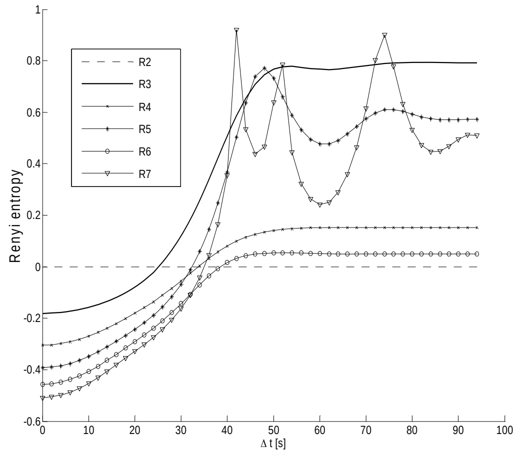

Figure 2. RE for different parameters α with respect to the time separation between the two Gabor logons.

Components separation in frequency and the RE

To show the effects of different choices of the parameter α on the RE for a signal whose components have a variable frequency separation, a signal with two linear frequency modulated (LFM) components has been chosen. Fig. 3 shows the WVD of the test signal for the component frequency separation Δf varying form 0 to $0.2\,Hz$.

From Fig. 4, showing the effects of different parameters α on the RE of the signal whose frequency separation between the components increases, it can be seen that for even α, Eq. (3) is not valid. Best results are obtained for $\alpha = 3$ since the oscillations (that are evident for $\alpha = 5$ and $\alpha = 7$) are almost non-existent, and the shortest transition period is achieved.

Our extensive simulations have shown that for the spectrogram (SP) and the Choi-Williams distribution (CWD) the choice of the parameter α brings minimal differences in the behavior of the RE for variable time and frequency separations of the signal components. In the case of $\alpha = 7$, a slightly shorter transition period has been observed for both the SP and the CWD, and thus in the rest of the paper the simulations including the RE of the SP and the CWD will be performed for $\alpha = 7$.

All previous works in the literature regarding the components counting using the RE imply the knowledge of the RE of at least one component [3]. This information, however, in practical situations is often unavailable. We next examine the effects of time duration and frequency bandwidth of signals on the RE of different TFDs, in order to find out if the number of signal components can be obtained from the RE of the TFD with no a priori information of the signal.

The component frequency bandwidth and the RE. The effects of different frequency bandwidths on the RE is illustrated on the examples of two signals A_1 and A_2, shown in Fig. 5 and 6, respectively. The signal A_1 is a one-component LFM whose frequency changes from 0.0 to 0.5 *Hz* with the time duration of 256 s. The signal A_2 is a cosine signal whose frequency is 0.25 *Hz* with the time duration of 256 s. The

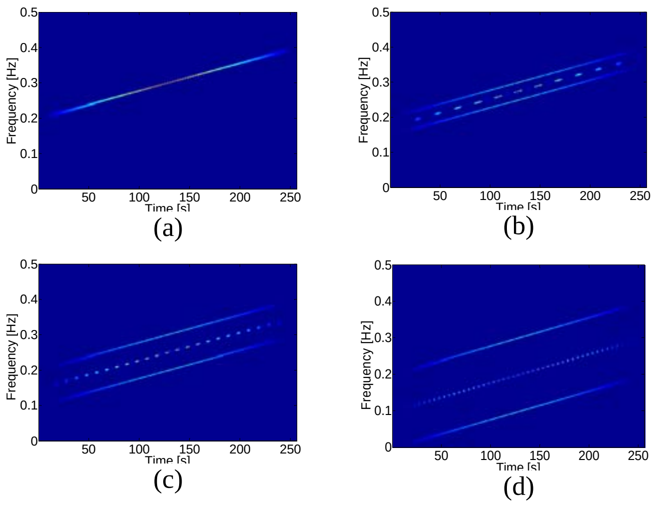

Figure 3. Separation of two LFM components in the TF plane: $\Delta f = 0$ Hz (a), $\Delta f = 0.05$ Hz (b), $\Delta f = 0.1$ Hz (c), $\Delta f = 0.2$ Hz (d).

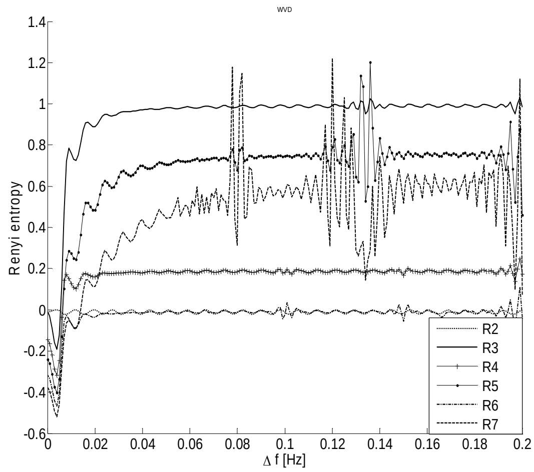

Figure 4. RE for different parameters α with respect to the time separation between the LFM components.

RE values for the two signals TFDs are show in Table 1.

	R_7 (SP)	R_7 (CWD)	R_3 (WVD)
A_1	3.9383	3.0479	0.0672
A_2	2.6686	2.0761	-0.0845

Table 1. The Renyi entropy of A_1 and A_2 for different TFDs

From Table 1, we can see that the bandwidth increase has the least influence on the RE of the WVD. Let us now take as an example the result obtained for the RE of the SP of the signal A_1, 3.938: from this value it can't be found out if the signal has one component with long or two components with short frequency bandwidths, since the RE of a signal with two components with constant FMs and with the same time duration achieves the value of 3.669. The CWD shows the same limitation. In the WVD case, however, the RE is close to zero, therefore we can say that the signal has one component, since in the case of a two-component signal the RE of the WVD is approximately one.

The component time duration and the RE. Untill now all the considered signals had the same time axis length of 256 s. We next examine how the changes in the time axis length (the signal measurement time), $M[s]$, affects the RE.

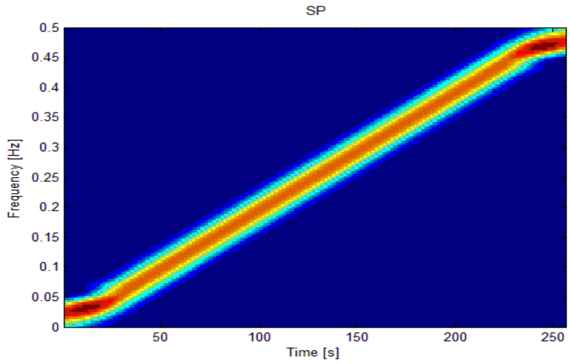

Figure 5. SP of the signal A_1.

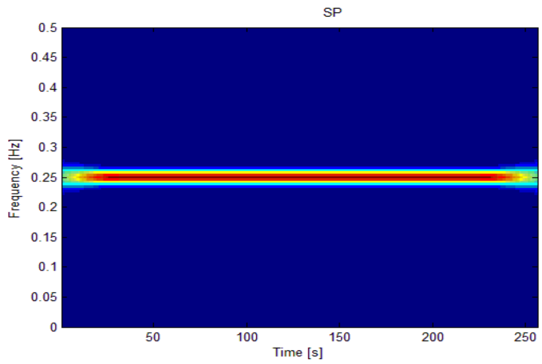

Figure 6. SP of the signal A_2.

Let our test signal be a Gabor logon with 32 *s* duration in time and let it be centered at 200 *s*. We evaluate the RE over a range of values of M for this test signal and the three TFDs. The results for the WVD, the SP, and the CWD are shown in Fig. 7. In the case of the WVD for $\alpha = 3$, the value of the RE stays unchanged, i.e. it is not a function of M. For the SP and the CWD, R_α is a nearly linear function of M. We need to mention that in the case of the CWD instead of $\alpha = 7$ we have used $\alpha = 3$ for faster simulation. The simulations have shown that the WVD is the least sensitive to variations in the signal duration.

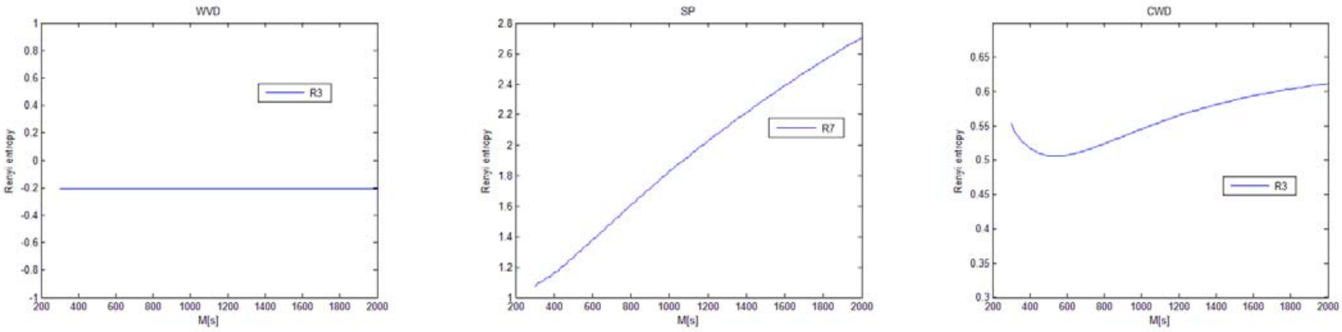

Fig. 7. RE as a function of the signal measurement time M for the WVD, SP and CWD od a Gabor logon.

From the results that have been presented in this paper we can conclude that due to its invariance to signal time duration and frequency bandwidth changes in the RE of the signal WVD, the WVD is the most suitable TFD for determining the number of components in the signal using the RE.

The RE of a signal class

A class represents a group of specific components. Fig. 8 and 9 illustrate a multicomponent signal and a class. If a Gabor logon and a cosine signal are a class, then the signal in Fig. 8 is composed of three such classes.

Let $R_\alpha(K_n)$ be the RE of n classes, $n \in Z^+$, and let $R_\alpha(K_1)$ be the RE of one class. Then the number of identical classes present in the signal is:

$$R_\alpha(K_n) - R_\alpha(K_1) = \log_2 n \tag{6}$$

where the *class* K_1 can be made of an arbitrary set of components.
For the class in Fig. 9 and the signal in Fig. 8, from Eq. (6) we have:
$R_\alpha(K_n) - R_\alpha(K_1) = 4.4012 - 2.8116 = \log_2 n \quad \Rightarrow \quad n = 3.01 \approx 3$
The number of classes will also be correctly detected if either the WVD or the CWD had been used.

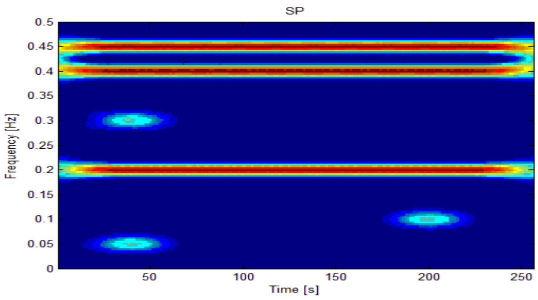

Fig. 8. SP of a multicomponent signal

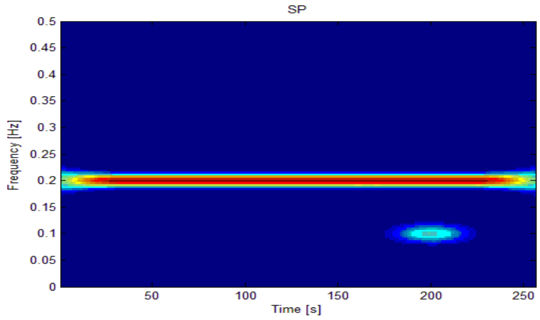

Fig. 9. SP of a signal class composed of two components

Conclusion

In this paper an analysis of the properties of the RE when used in determining the complexity of a multicomponent nonstationary signal, has been presented. The optimal parameter α of the Rényi entropy has been determined for each of the considered time-frequency distributions (the WVD, the SP, the CWD). By analyzing the effects of time duration and frequency bandwidth of the signals on the RE, it has been concluded that the WVD is the most suitable among the tested distributions for determining the component number of a signal from the RE of its TFD. We have also shown that the content of a multicomponent signal can be estimated by the RE not only with regard to single components, but a *class* of components as well.

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