Using Gradient Orientation to Improve Least Squares Line Fitting

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Abstract

Straight line fitting is an important problem in computer and robot vision. We propose a novel method for least squares line fitting that uses both the point coordinates and the local gradient orientation to fit an optimal line by minimizing the proposed algebraic distance.

The proposed inclusion of gradient orientation offers several advantages:

(a) one data point is sufficient for the line fit,

(b) for the same number of points the fit is more precise due to inclusion of gradient orientation, and

(c) outliers can be rejected based on the gradient orientation or the distance to line.

1. Introduction

The least squares approach is commonly used to solve the straight line fitting problem [Haralick and Shapiro, 1992;Hornberg, 2011]. OpenCV [OpenCV, 2014], a computer vision software library, offers a rich set of line fitting procedures based on minimizing L1, L2 or modified orthogonal distances.

Most of currently used least-squares fitting procedures take only *point coordinates* x and y as input.

In a typical computer and robot vision application a line is fitted to an edge or to a ridge in two steps:

(a) extraction of point coordinates x and y using an edge or a ridge detector and

(b) application of a straight line fitting method.

Both edge and ridge detectors can output additional information: *the orientation of the edge* (gradient direction) or *the direction of the ridge*.

This information is usable in line fitting problem as it defines the line direction or the line normal, depending on the type of detector used.

The use of gradient orientation was first suggested for use in line extraction tasks in [Burns et al., 1986] where the gradient orientation is used to group pixels into compact line-support regions. This approach was extended in [Kahn et al., 1990; von Gioi et al., 2010; Pătrăucean et al., 2012] to improve the speed and to fully automatize the choice of parameters. However, the possibility of using the gradient orientation to improve the least squares line fitting was not investigated.

2. Line Fitting

A straight line is represented by a polynomial

$$F(\mathbf{a},\mathbf{x}) = \mathbf{a}\cdot\mathbf{x} = ax + by + c = 0,$$

where $\bar{\imath}a + \bar{\jmath}b = \bar{\imath}\cos\theta + \bar{\jmath}\sin\theta$ is the unit normal vector and c is the distance to the origin.

Point coordinates $\mathbf{x}_i$ are extracted from an edge map [Canny, 1986; Lanser and Eckstein, 1992]. Additional output of an edge detector is the gradient vector which is orthogonal to the local edge direction, so in straight line fitting problems its orientation θ defines the straight line normal.

For fitting a straight line to points $\mathbf{x}_k$ using a gradient orientation θ_k we propose the objective function

$$\sum_{k=1}^{N} w_{0,k}F^2(\mathbf{a},\mathbf{x}_k) + w_{1,k}(a-\cos\theta_k)^2 + w_{2,k}(b-\sin\theta_k)^2,$$

subject to the quadratic equality constraint

$$a^2 + b^2 = 1,$$

which forces the unit normal vector to be directly encoded in the parameters a and b so the error of fitting to the image gradient is fully contained in the proposed terms $a-\cos\theta_k$ and $b-\sin\theta_k$.

The solution that minimizes the objective function is obtained using the Lagrange multipliers yielding the system

$$\mathbf{S}\boldsymbol{\alpha} = \lambda\mathbf{C}\boldsymbol{\alpha} + \mathbf{d}$$
$$\boldsymbol{\alpha}^T\mathbf{C}\boldsymbol{\alpha} = 1$$

where $\boldsymbol{\alpha} = [a \quad b \quad c]^T$ is the vector of line parameters,

$$\mathbf{d} = \left[\textstyle\sum_k w_{1,k}\cos\theta_k \quad \sum_k w_{2,k}\sin\theta_k \quad 0\right]^T$$

is the offset vector (sum of gradient direction vector components),

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

is the constraint matrix, and

$$\mathbf{S} = \begin{bmatrix} \sum_k w_{0,k}x_k^2 + w_{1,k} & \sum_k w_{0,k}x_ky_k & \sum_k w_{0,k}x_k \\ \sum_k w_{0,k}x_ky_k & \sum_k w_{0,k}y_k^2 + w_{2,k} & \sum_k w_{0,k}y_k \\ \sum_k w_{0,k}x_k & \sum_k w_{0,k}y_k & \sum_k w_{0,k} \end{bmatrix},$$

is the modified scatter matrix. Equations are solved numerically (Levenberg-Marquardt).

3. Weights

The proposed term $(a-\cos\theta_k)^2 + (b-\sin\theta_k)^2$ encodes the angular difference $\Delta\theta = \theta - \theta_k$ (corrected for wrapping) via the monotonic mapping

$$\Delta\theta = 2\,\mathrm{asin}\left(\frac{1}{2}\sqrt{(a-\cos\theta_k)^2 + (b-\sin\theta_k)^2}\right)$$

that preserves the ordering and does not introduce bias.

The angular differences $\Delta\theta = \theta - \theta_k$ (corrected for wrapping) are limited to the $[-\pi, \pi]$ interval and $(a-\cos\theta_k)^2 + (b-\sin\theta_k)^2$ is therefore limited to the $[0, 4]$ interval.

The distance term $F^2(\mathbf{a},\mathbf{x}_k)$ is not bound in the same way.

To make the contribution of both distance and angular terms comparable the weights $w_{j,k}$ must be adjusted. W.l.o.g. let $w_{0,k} = 1$ and $w_{1,k} = w_{2,k} = w$, so the objective function becomes

$$\sum_{k=1}^{N} F^2(\mathbf{a},\mathbf{x}_k) + w((a-\cos\theta_k)^2 + (b-\sin\theta_k)^2).$$

The weight w should be chosen so two distance and angular terms are comparable.

The choice will be application specific and will depend on the units used to measure the distance as the proposed angular error is always bound to $[0, 4]$ interval.

4. Applications in RANSAC

The proposed fitting method is easily used in RANSAC schemes [Fischler and Bolles, 1981]. A typical RANSAC scheme is comprised of the following steps:

1. pick a (minimal) random sample from the data required for fitting and then fit a model to the minimal sample;
2. classify all points as inliers (consensus set) and outliers, based on the proximity to the model;
3. if the consensus set is large enough re-fit the model using all inlier points and establish some quality of fit measure (normally the size of the consensus set), if not, discard the random sample as not good enough; and
4. repeat steps 1-3 sufficient number of times and pick a model with the best quality of fit as the final solution.

There are several advantages the proposed straight line fitting scheme offers:

(a) the number of points required for the initial straight line fit is reduced from two to one,

(b) the selection criteria for inlier and outlier points is based on both the geometric and the angular distance to the line, and

(c) given the same number of points the proposed fit is better.

5. MATLAB Implementation

```
% x, y, theta are colum vectors, w is weight for the angular component.
function a = fit_line2(x, y, theta, w)
% Build 3x3 scatter matrix.
S = zeros(3);
S(1,1) = sum(x.^2); S(2,1) = sum(x.*y); S(3,1) = sum(x);
S(1,2) = S(2,1);    S(2,2) = sum(y.^2); S(3,2) = sum(y);
S(1,3) = S(3,1);    S(2,3) = S(3,2);    S(3,3) = numel(x);
% Build 3x3 constraint matrix.
C = zeros(3);
C(1,1) = 1; C(2,2) = 1;
% Solve generalized eigensystem.
[gevec, geval] = eig(S, C, 'qz');
for k = 1 : 3
    mu(k) = 1 ./ sqrt(gevec(:,k).' * C * gevec(:,k));
    u(:,k) = mu(k) * gevec(:,k);
    E(k) = u(:,k).' * S * u(:,k);
end
[minE, idx] = min(abs(E));
% Initial solution is normal straight line fit.
x0 = [u(:,idx); geval(idx,idx)];
% Build offset vector.
d = [sum(w.*cos(theta)) sum(w.*sin(theta)) 0].';
% Modify scatter matrix.
S(1,1) = S(1,1) + sum(w .* ones(size(theta)));
S(2,2) = S(2,2) + sum(w .* ones(size(theta)));
% Build objective function.
eqs = @(x) [S*x(1:3) - d - x(4)*C*x(1:3); x(1).^2 + x(2).^2 - 1];
% Solve and return.
[x1, qof] = fsolve(eqs, x0, optimset('Display','Off'));
a = x1(1:3);
```

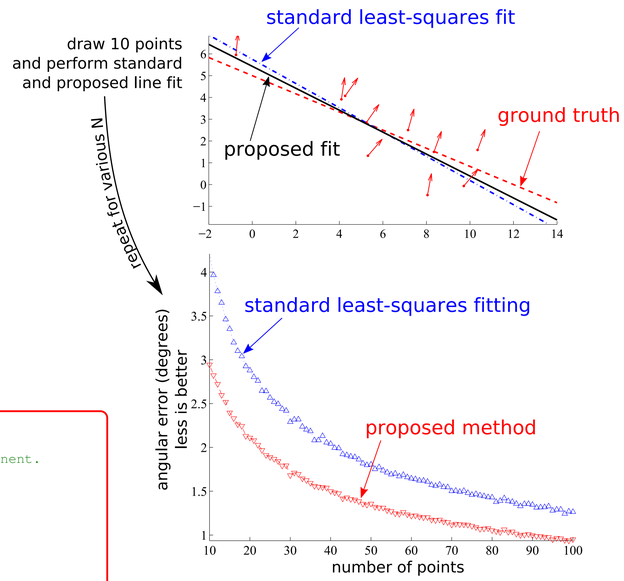
6. Experiment A

A qualitative comparison of proposed fitting method to the standard least squares fitting method using the Matlab code and $w = 8$. From the line

$$0 = \frac{5}{13}x + \frac{12}{13}y - \frac{60}{13} \approx 0.3846x + 0.9231y - 4.6154$$

we randomly draw N points, where N goes from 10 to 100. We add Gaussian noise $\mathcal{N}(0, 1)$ to x and y coordinates while the gradient is drawn from the von Mieses distribution (normal distribution on a circle) using a variance of 17°. For each N the random draw using aforementioned variances of 1 for point coordinates and of 17° for gradient orientation was repeated 5000 times.

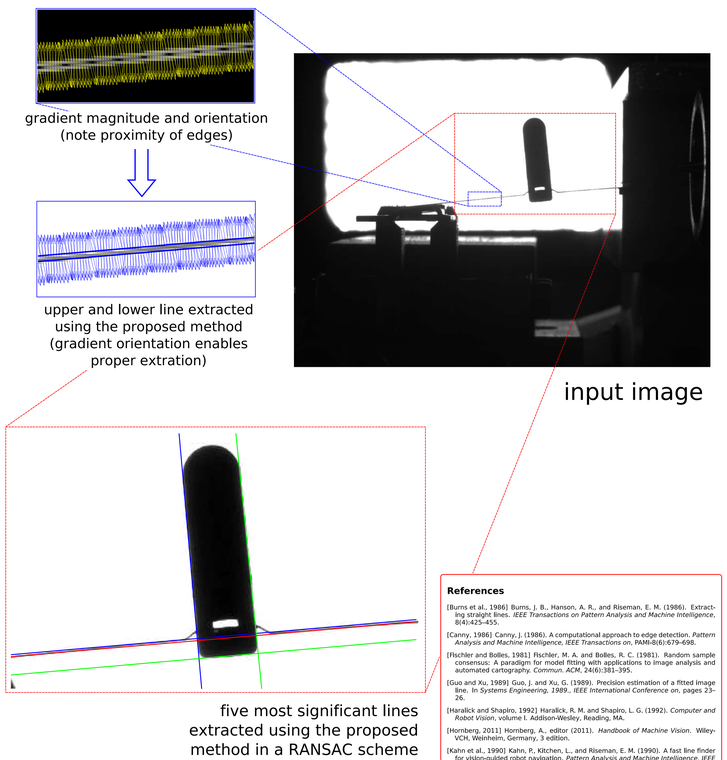
Due to inclusion of gradient orientation, the proposed method outperforms the standard least squares fitting method and gives a better slope estimate.



7. Experiment B

Application of the proposed method to a problem from industry: visual quality inspection. The problem is to fit upper and lower lines to a thin structure shown within inner red region-of-interest (ROI) rectangle. We use an iterative RANSAC scheme (fit the best line, remove consensus points from the set, and repeat) to find all lines. We rely on the gradient orientation to separate between the upper and the lower edges, thus avoiding mixing of the data.

The five most significant extracted lines are shown; note that upper and lower line that would have been quite difficult to fit properly if the gradient orientation information was not included into the fit were correctly extracted.



Conclusion

We have modified the conventional least squares line fitting method to incorporate the orientation of the image gradient thus obtaining the novel straight line estimator that offers several several advantages:

(a) one data point is sufficient for the line fit,

(b) for the same number of points the fit is more precise due to inclusion of the gradient orientation, and

(c) in addition to simple distance measures outliers can be rejected based on the gradient orientation.

The proposed fitting method is especially suitable in applications where the line slope must be extracted from the image data due to improved precision compared to the standard fitting as the gradient orientation information is included.

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