A robust separable image denoising based on relative intersection of confidence intervals rule

Damir Seršić and Ana Sović (ana.sovic@fer.hr)
University of Zagreb, Faculty of Electrical Engineering and Computing, Croatia
Overview

- Problem
- Intersection of confidence intervals rule
- Relative intersection of confidence intervals rule
- Separable image denoising
- Examples
- Conclusion
Problem

- Input signal (image) corrupted with noise
  \[ y(n) = x(n) + w(n) \]
- Goal: find a good estimate \( \tilde{x}(n) \) of input signal that accurately restores smooth parts and preserves its edges and discontinuities
Moving average

- Zero-order estimate of the input signal

\[ \bar{x}_{h_k}(n) = \frac{1}{h_k} \sum_{i=\langle h_k \rangle} y(i) \]

- Large windows \( h_k \)
  - good estimates of low frequency components
  - unwanted smoothing of jumps and edges

- Optimal window length \( h^* \) at some sample \( n \) → the smoothest zero-order estimate that does not span across the edges
Intersection of confidence intervals

- Growing set of estimators of window lengths

\[ h_1 < h_2 < \cdots < h_k < \cdots < h_K \]

- Increase \( h_k \) until we reach the optimal window length \( h^* \) for each observed sample \( n \)
Intersection of confidence intervals

- **Confidence interval**
  \[ D_k(n) = [L_k, U_k], \quad 1 \leq k \leq K \]

- **Lower and upper limits**
  \[
  L_k = \bar{x}_{h_k}(n) - \Gamma \cdot \sigma_{h_k}(n) \\
  U_k = \bar{x}_{h_k}(n) + \Gamma \cdot \sigma_{h_k}(n)
  \]

- **The smallest upper and the largest lower confidence interval limits**
  \[
  \bar{L}_k(n) = \max_{i=1,\ldots,k} L_i(n) \\
  \underline{U}_k(n) = \min_{i=1,\ldots,k} U_i(n)
  \]

\[ \sigma_{h_k}(n) = \frac{\sigma_w}{\sqrt{h_k}} \]
ICI rule

- Optimal window length $h^+ = h_\kappa$ is the largest window length for which the condition

$$\bar{L}_\kappa(n) \leq \underline{U}_\kappa(n)$$

is satisfied

- Close to ideal $h^*$

- Repeat for each sample of $y(n)$
Relative intersection of confidence intervals (RICI)

- chosen window length is the largest \( h^+ = h_k \) for which is still satisfied

\[
\frac{U_k(n) - \bar{L}_k(n)}{U_k(n) - L_k(n)} \geq r_c
\]
Estimate of the input signal

\[ \bar{x}_{h^+}(n) = \text{mean} \{ y(n), \ldots, y(n + h^+(n) - 2), y(n + h^+(n) - 1) \} \]

- In asymptotical sense → mean can be replaced by median

\[ \bar{x}_{h^+}(n) = \text{median} \{ y(n), \ldots, y(n + h^+(n) - 2), y(n + h^+(n) - 1) \} \]
Why median

- results are more robust (even if the window lengths were not perfectly determined)
- more accurate results for almost any noise distribution
- significantly less sensitive to choice of parameters
- less sensitive to outliers
Image denoising algorithm – 1st step

1. For each pixel $y(n,m)$ observe its ROW
2. Find the widest window length on the left side and on the right side from $y(n,m)$ using ICI or RICI rule
3. Join all pixels on the intervals – use median – intermediate image $\bar{x}_r(n, m)$

4. For each pixel $\bar{x}_r(n, m)$ observe its COLUMN
5. Find the widest window length on the upper side and on the down side from $\bar{x}_r(n, m)$ using ICI or RICI rule
6. Join all pixels on the intervals – use median – intermediate image $\bar{x}_{rc}(n, m)$
Image denoising algorithm – 2nd step

1. For each pixel $y(n,m)$ observe its COLUMN
2. Find the widest window length on the upper side and on the down side from $y(n,m)$ using ICI or RICI rule
3. Join all pixels on the intervals – use median – intermediate image $\tilde{x}_c(n,m)$

4. For each pixel $\tilde{x}_c(n,m)$ observe its ROW
5. Find the widest window length on the left side and on the right side from $\tilde{x}_c(n,m)$ using ICI or RICI rule
6. Join all pixels on the intervals – use median – intermediate image $\tilde{x}_{cr}(n,m)$
Image denoising algorithm – 3rd step

- Combine estimated images $\bar{x}_{rc}(n,m)$ and $\bar{x}_{cr}(n,m)$ to the final denoised result

- **MRICI method with fixed weights**
  \[
  \bar{x}(n,m) = \frac{\bar{x}_{rc}(n,m) + \bar{x}_{cr}(n,m)}{2}
  \]

- **MRICI method with the variable weights**
  \[
  \omega_{rc}(n,m) = h_{rcL}^+(n,m) + h_{rcR}^+(n,m) + h_{rcU}^+(n,m) + h_{rcD}^+(n,m)
  \]
  \[
  \omega_{cr}(n,m) = h_{crL}^+(n,m) + h_{crR}^+(n,m) + h_{crU}^+(n,m) + h_{crD}^+(n,m)
  \]
  \[
  \bar{x}(n,m) = \frac{\omega_{rc}(n,m) \cdot \bar{x}_{rc}(n,m) + \omega_{cr}(n,m) \cdot \bar{x}_{cr}(n,m)}{\omega_{rc}(n,m) + \omega_{cr}(n,m)}
  \]
### PSNR for 30 realizations of the test image

<table>
<thead>
<tr>
<th></th>
<th>Gaussian noise, dB</th>
<th>Laplacian noise, dB</th>
<th>Binomial noise, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>noisy-image</strong></td>
<td>28.1372</td>
<td>28.1263</td>
<td>27.7333</td>
</tr>
<tr>
<td><strong>ICI, fixed weight</strong></td>
<td>19.5834</td>
<td>19.5659</td>
<td>19.7396</td>
</tr>
<tr>
<td><strong>ICI, variable weight</strong></td>
<td>19.5706</td>
<td>19.5530</td>
<td>19.7269</td>
</tr>
<tr>
<td><strong>MICI, fixed weight</strong></td>
<td>28.1511</td>
<td>28.1373</td>
<td>27.7886</td>
</tr>
<tr>
<td><strong>MICI, variable weight</strong></td>
<td>28.1523</td>
<td>28.1405</td>
<td>27.7762</td>
</tr>
<tr>
<td><strong>RICI, fixed weight</strong></td>
<td>40.8962</td>
<td>38.9303</td>
<td>36.3624</td>
</tr>
<tr>
<td><strong>RICI, variable weight</strong></td>
<td>40.7430</td>
<td>39.0041</td>
<td>36.2947</td>
</tr>
<tr>
<td><strong>MRICI, fixed weight</strong></td>
<td><strong>42.1992</strong></td>
<td>40.1020</td>
<td><strong>42.6739</strong></td>
</tr>
<tr>
<td><strong>MRICI, variable weight</strong></td>
<td>41.8532</td>
<td><strong>40.1671</strong></td>
<td>42.3026</td>
</tr>
<tr>
<td><strong>anisotropic LPA-ICI</strong></td>
<td>39.9478</td>
<td>37.3952</td>
<td>35.5632</td>
</tr>
<tr>
<td><strong>Haar wavelet</strong></td>
<td>40.4542</td>
<td>39.7961</td>
<td>35.9131</td>
</tr>
</tbody>
</table>
Image + noise (different noise deviations)

(a) Gaussian noise

(b) Laplacian noise

(c) Binomial noise

- LPA-ICI
- Haar
- RICI
- MRICI
Conclusion

- novel method for image denoising based on the relative intersection of confidence interval rule
- separable three steps method
- median based estimators

- smooth parts are accurately restored
- edges and discontinuities are reserved
Thank you for your attention!